

Light pulse slowing down using backward-wave four-wave mixing

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ABSTRACT

The nonlinear phase shift per unit length acquired by the transmitted and phase conjugate waves in the backward-wave four-wave mixing may be considered as the phase shift obtained because of an effective refractive index. Consequently, a spectrum of the nonlinear phase shift in non-degenerate frequency interaction corresponds to natural dispersion and such interaction may be used for temporal manipulation of light pulses. Analysis of the narrow spectrum of the nonlinear phase shift shows high potential of the backward-wave four-wave for slowing down of light pulses. The slowing down is achieved using backward-wave four-wave mixing in media with local and nonlocal response. The delay and shape transformation of output pulses are studied and compared for the transmitted and phase conjugate channels. It is shown that the phase conjugate pulse achieves a longer delay under typical experimental conditions.

Keywords: slow light, pulse propagation, backward-wave four-wave mixing, phase conjugation, dynamic gratings, dynamic holography

1. INTRODUCTION

The propagation of light pulses in a dispersive medium has been studied for more than a century^{1,2}. The study in this topic got a powerful impulse when a strong dispersion related to the quantum effect of Electromagnetically Induced Transparency allowed remarkable slowing down of light pulses down to group velocity $v_{gr} = 32 \text{ m/s}$ ³. In this case a spatially uniform but frequency-dependent refractive index n , i.e., a material dispersion $dn/d\omega$, is important for slowing down of light pulses⁴. As the dispersion is pronounced in the vicinity of absorption or gain resonances the slow light effects are also pronounced within such resonances related to material properties.

Resonances in different three-dimensional structures may be used for light pulses slowing down, too. Such “structural slow light”⁴ is considered when the propagation of light pulse is significantly modified by non uniform in space optical properties in structures like photonic crystals or fiber Bragg gratings.

At the same time, spatial modulation of optical properties can be created by optical waves mixed in a nonlinear medium. This is the case of dynamic holograms recording. The narrow Bragg resonance of such dynamic index gratings allowed deceleration of the group velocity down to $v_{gr} = 0.025 \text{ cm/s}$ ⁵. In this case the nonlinear phase shift per unit length obtained by optical waves in the nonlinear interaction may be considered as an effective refractive index. Consequently, a narrow spectrum of the nonlinear phase shift in non-degenerate in frequency interaction corresponds to natural dispersion. Therefore such interaction may be used for temporal manipulation of light pulses.

The backward-wave four-wave mixing (BWFWM) is one of the most common schemes in dynamic holography. It is particularly interesting for slowing down of light pulses⁶⁻⁹ because BWFWM can be realized using different physical effects in a large variety of materials. Therefore a proper selection of a material with a required response time, spectral sensitivity range, dynamic intensity range, etc., allows the achievement of a necessary pulse delay for a light pulse with a required wavelength and within necessary intensity level. Other characteristics of the decelerated pulses, such as the pulse amplification and the nonlinear shape transformation may also be significantly optimized.

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In the present work the studies of slowing down of light pulses with BWFWM are summarized. The spectral profiles of the amplitude and nonlinear phase shift of the complex phase conjugate reflectivity and transmittance are analyzed for the purpose of qualifying the BWFWM for light pulse slowing down. The performances of nonlocal and local dynamic holograms are compared using different photorefractive crystals. The delay and shape transformation of the transmitted and phase conjugate output pulses are studied for various experimental conditions. It is shown that the different origin of the phase conjugate and transmitted outputs results in much longer delay of the phase conjugate pulses for short probe pulses with half-width smaller than the response time of the medium. The agreement of the experimental results with numerical calculations confirms the validity of the previously developed theoretical model.

2. BWFWM FOR SLOWING DOWN OF LIGHT PULSES

When an optical nonlinear medium is pumped by two counter propagating waves 1 and 2 as it is shown in Fig. 1a, the probe wave 4 sent to the illuminated region interacts with the pumps and gives rise to the appearance of a back-propagating wave 3. The wave 3 is the phase conjugate replica of the signal wave 4 for pump waves 1 and 2 with mutually conjugated wavefronts¹⁰⁻¹². For the first time this effect was reported for Kerr-medium¹², which has the local response from the general point of view. This effect has been observed also in media with different optical nonlinearities and is denoted as the backward-wave four-wave mixing.

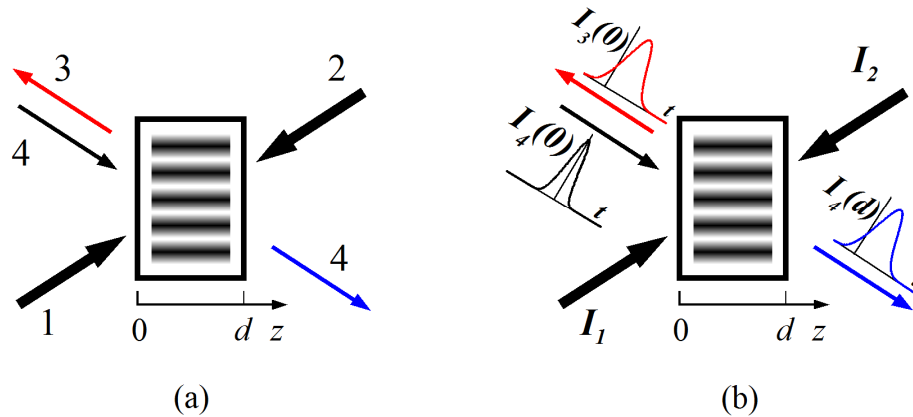


Figure 1. Schematic representations: (a) – BWFWM configuration, (b) – input/output pulses in the experiments on light pulses slowing down.

In what follows we consider the recording of transmission gratings only, i.e., the dynamic hologram is recorded by waves 1 and 4 while the phase conjugate wave 3 appears as a result of the diffraction of the pump 2 from this hologram. Therefore the wave 3 appearing at $z = d$ and propagating in the medium records a second grating with the co-propagating pump 2. Within the common approximation of non-absorbing crystal and undepleted pumps the theory¹³ predicts qualitatively similar results on phase conjugation (but with modified value of the coupling constant) for the case of recording of only reflection or both, transmission and reflection, gratings.

The phase and the group velocity are connected by the relation¹⁴

$$v_{gr}(\omega) = \frac{c}{n + \omega \frac{dn(\omega)}{d\omega}}, \quad (1)$$

where n is an unperturbed value of the refractive index and ω is the angular frequency of the light. In the case of nonlinear interaction the phase conjugate wave acquires a phase shift, which may be considered in terms of an effective refractive index for this wave. Consequently the group velocity may be written

$$v_{gr}(\Delta\omega) = \frac{c}{n + c \frac{d\phi'(\Delta\omega)}{d\Delta\omega}} \approx \frac{1}{d\phi'(\Delta\omega)/d\Delta\omega}, \quad (2)$$

where $\phi'(\Delta\omega)$ is the phase shift per unit length and $\Delta\omega$ is the frequency detuning from the Bragg resonance of the probe wave. It is clear from Eq. (2) that a positive derivative $d\phi'(\Delta\omega)/d\Delta\omega$ is a sign of a medium which may ensure a slowing down of light pulses due to nonlinear interaction.

The steady state complex phase conjugate reflectivity^{13,15} in the case of nearly degenerate in frequency BWFWM takes the form of spectral response

$$\rho(\Delta\omega) = \frac{A_3(0, \Delta\omega)}{A_4^*(0, \Delta\omega)} = \frac{\sinh\left(\frac{\Gamma_\omega d}{2}\right)}{\cosh\left(\frac{\Gamma_\omega d}{2} + \frac{\ln r}{2}\right)} = |\rho(\Delta\omega)| \exp[i\varphi_3(\Delta\omega)], \quad (3)$$

where $\Gamma_\omega = \gamma/(1 - i\Delta\omega\tau)$ is the rate coefficient for the wave amplitudes, γ is the coupling constant, which is generally complex, d is the interaction length, τ is the response time, $r = I_2(d)/I_1(0)$ is the pump intensity ratio. The asterisk stands for complex conjugation.

The spectral profiles of the amplitude and the phase of the phase-conjugate reflectivity calculated from Eq. (3) for nonlocal grating recorded in a photorefractive crystal with coupling constant $\gamma d = -1$ and for $r = 1$ are shown in Fig. 2a and Fig. 2b, respectively. The amplitude exhibits a Lorentz-like profile typical for isolated oscillator. The negative sign of the coupling constant corresponds to the experimental geometry where the transmitted wave 4 is amplified while the phase-conjugate beam 3 is attenuated.

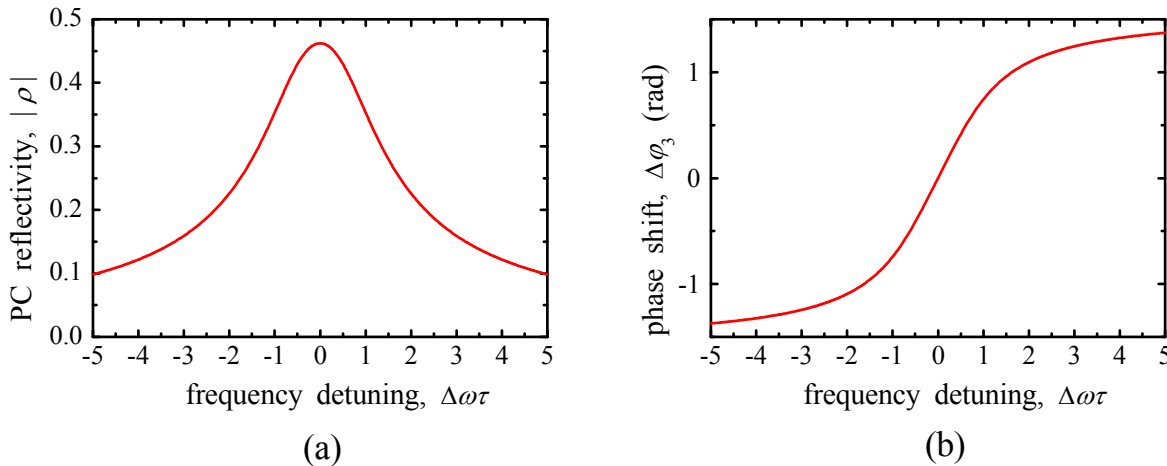


Figure 2. Spectral profiles of the amplitude (a) and phase shift (b) of the phase-conjugate reflectivity calculated from Eq. (3) for nonlocal grating with coupling constant $\gamma d = -1$ and $r = 1$

In addition to the phase conjugate wave $A_3(0)$ there is a second output in BWFWM. This is the transmitted wave $A_4(d)$. If the phase conjugate wave $A_3(0)$ consists of only one component diffracted from pumps, the main difference of the wave $A_4(d)$ is that it includes two components: one component that passes through the medium in the direction of wave $A_4(d)$, and a second component diffracted from the pump A_1 . The normalized transmittance for the wave 4 is given by

$$T(\Delta\omega) = \frac{A_4(d, \Delta\omega)}{A_4^*(0, \Delta\omega)} = \frac{\exp\left(\frac{\Gamma_\omega d}{2}\right) \cosh\left(\frac{\ln r}{2}\right)}{\cosh\left(\frac{\Gamma_\omega d}{2} + \frac{\ln r}{2}\right)} = |T(\Delta\omega)| \exp[i\varphi_4(\Delta\omega)]. \quad (4)$$

The curve of the nonlinear phase shift in Fig. 2b has a positive slope in the vicinity of zero frequency detuning. Therefore the phase conjugate pulse should be delayed as compared to the input pulse, if the spectral content of the last belongs to the nearly linear growth of the effective refractive index in the nonlinear dispersion curve shown in Fig. 2b. The spectral profiles of the amplitude and nonlinear phase shift of the transmitted wave generally are similar to that for the phase conjugate wave in the case of nonlocal response with relatively small coupling constant⁸. However the corresponding spectra for nonlocal response differ dramatically for the transmitted and phase conjugate pulses in the case of local response⁹. At the same time there is still a rather steep increase of the nonlinear phase shift $d\varphi(\Delta\omega)/d\Delta\omega$ in the vicinity of zero frequency detuning. Therefore the slowing down of light pulses is possible with local dynamic holograms, too. The experimental results and numerical calculations confirm this qualitative analysis.

3. EXPERIMENTAL RESULTS

Different photorefractive crystals are used in the experiment. To study the grating with local response a dc field is applied to the crystal, while no field is used in the experiments with nonlocal gratings. Two cw pump beams I_1 and I_2 enter the sample as shown in Fig.1. The Gaussian shaped pulse is tailored by an electro-optic modulator in a signal beam, with intensity $I_4(d=0,t) = I_4^0 \exp(-t^2/t_0^2)$, where t_0 is the $1/e$ half-width of the pulse intensity. The temporal envelopes of the input $I_4(0,t)$, transmitted $I_4(d,t)$ and phase-conjugate $I_3(0,t)$ signals are recorded for different input pulse durations and various experimental conditions. For the characterization of the slowing down of the light pulses we use a time interval Δt between the maximum of the input pulse intensity and the maximum of the output pulse $I_3(0,t)$ or $I_4(d,t)$. For the evaluation of the transformation of the output pulses we use the output pulse half-width w at the intensity level $1/e$ normalized to the input pulse half-width t_0 .

The output pulses $I_4(d,t)$ and phase-conjugate $I_3(0,t)$ are shown in Fig. 3 measured for the SBN crystal with coupling constant $|\gamma d| = 1.7$ and $\tau = 1.4$ s for the input pulse with half width $t_0 = 1.6$ s and beam ratio $\beta = 0.18$. It is obvious that the phase conjugate pulse is delayed for a longer time interval as compared to the transmitted pulse. This difference is much larger for shorter pulses⁶⁻⁹ with duration shorter than the response time of the material τ .

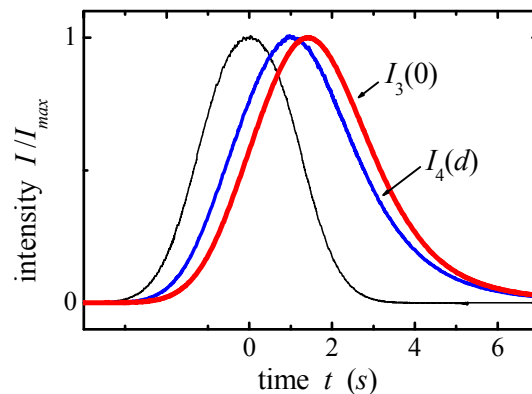


Figure 3. Temporal variation of the intensities of input pulse $I_4(0)$ (thin line), transmitted $I_4(d)$ and phase-conjugate $I_3(0)$ pulses measured for the SBN crystal with $|\gamma d| = 1.7$ and $\tau = 1.4$ s for input pulse half-width $t_0 = 1.6$ s and beam ratio $\beta = 0.18$.

The complex characteristic of the pulse delay is the fractional delay. The fractional delay calculated as the ratio of pulse delay Δt and the output pulse half-width w is shown in Fig. 4 as a function of the input pulse half-width for SBN crystal with nonlocal response with coupling constant $|\gamma d| = 1.7$ and $\tau = 1.4$ s (Fig. 4a) and for dc-biased $\text{Bi}_{12}\text{TiO}_{20}$ crystal exhibiting local response with coupling constant $|\gamma d| = 1.15$ and $\tau = 0.8$ s (Fig. 4b). The squares represent data for the phase conjugate pulse while the opened diamonds correspond to the transmitted beam. Solid lines represent calculations according to the theory¹⁶ calculated with the coupling constant and response time evaluated from independent experiments. It is obvious that the phase conjugate pulse exhibits the longer fractional delay as for local as well as for

nonlocal response. Such behavior is observed with change of different experimental conditions such as intensity ratio, input pulse duration and coupling constant. That is why the use of BWFWM looks much more preferable for the slowing down of light pulses as compared to the two-beam coupling scheme.

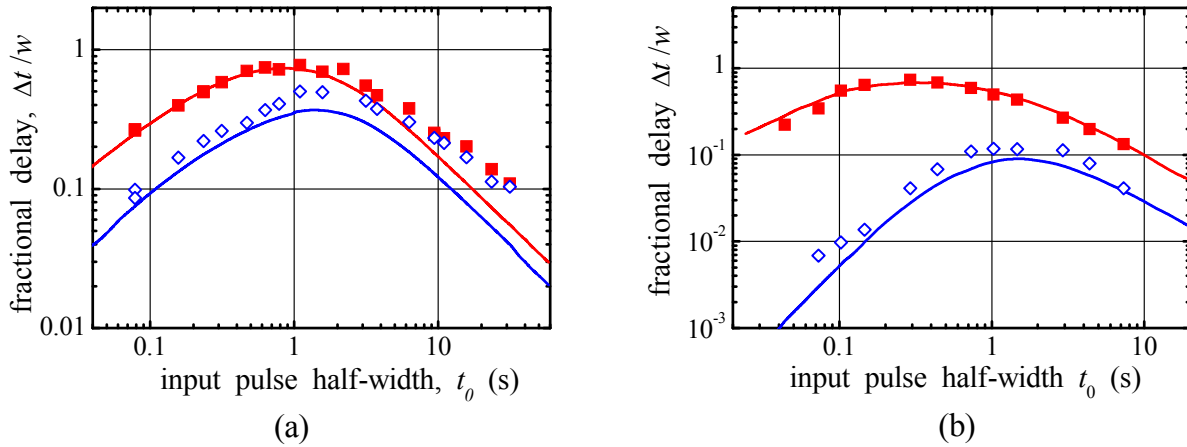


Figure 4. Fractional delay $\Delta t/w$ as a function of the input pulse half-width, (a) – SBN crystal exhibiting nonlocal response with coupling constant $|\gamma d|=1.7$, $\tau=1.4$ s, measured for $r=0.18$; (b) – dc-biased $\text{Bi}_{12}\text{TiO}_{20}$ exhibiting local response with coupling constant $|\gamma d|=1.15$, $\tau=0.8$ s, measured for $r=1.6$; the squares represent data for the phase conjugate pulse while the opened diamonds correspond to the transmitted beam.

4. CONCLUSIONS

A slowing down of light pulses is achieved using BWFWM in a photorefractive crystal exhibiting nonlocal and local response. The delay and broadening of the output pulse for the transmitted and phase conjugate beams are studied and compared for different input pulse durations and various experimental conditions. We demonstrated that the four-wave mixing process ensures a larger delay of short pulses compared to the photorefractive two-beam coupling and guarantees the elimination of a precursor, which is a principal drawback for the slowing down of short pulses in the two-beam coupling with nonlocal response. The agreement of the experimental results with the theoretical model confirms the previous theoretical study¹⁶. The relatively small delay time achieved in the experiment may be increased with larger coupling constant, as the theory predicts.

The photorefractive crystals, which we use in our demonstrational experiments, are quite slow at low intensities. The response time of photorefractive crystals is inversely proportional to the light intensity and changes from the 10^2 seconds range for ferroelectrics at low power excitation to the sub-nanosecond range for semiconductors at high power pulsed excitation¹⁷. Therefore the slowing down of pulses in sub-nanosecond range may be achieved with fast photorefractive semiconductors.

Generally, the backward wave four-wave mixing can be realized via different physical phenomena like, for example, thermal gratings, optical Kerr effect, gratings in gain media, etc. Therefore the slowing down characteristics, such as dynamic range, pulse delay and nonlinear broadening, will be determined by the response time of the medium, the type of the response, such as local or nonlocal, and by the dispersive properties. That is why the dynamic range and characteristics of slowed down pulses may be significantly extended. For example, photorefractive and free-carrier gratings in semiconductors at high intensity excitation or dynamic population gratings allow four-wave mixing based slow light with much shorter light pulses than we explore with our modeling media.

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