Four-wave-mixing coherent oscillator with frequency shifted feedback and misaligned pump waves

Riadh Rebhi,[1] Pierre Mathey,[1,*] Hans Rudolf Jauslin,[1] Serguey Odoulov,[2] Gary Cook,[3,4] and Dean R. Evans[3]

[1]*Institut Carnot de Bourgogne, UMR 5209 CNRS-Université de Bourgogne, 9 Avenue Alain Savary, BP 47870, 21078 Dijon CEDEX, France*

[2]*Institute of Physics, National Academy of Sciences, 03 650 Kyiv, Ukraine*

[3]*Air Force Research Laboratory, Materials and Manufacturing Directorate, Wright-Patterson Air Force Base, Ohio 45433, USA*

[4]*Universal Technology Corporation, 1270 N. Fairfield Road, Dayton, Ohio, 45432, USA*

**Corresponding author: pmathey@u-bourgogne.fr*

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The effect of the pump waves misalignment on the oscillation spectra and oscillation intensity of a semilinear photorefractive oscillator is studied numerically and compared with the results of the experiment performed with a $KNbO_3$:Fe,Ag crystal. © 2009 Optical Society of America

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The semilinear oscillator with two pump waves is the simplest four-wave-mixing oscillator with a phase conjugate mirror, first considered 30 years ago [1]. The cavity is formed by one conventional mirror and a phase conjugate mirror, which presents interest for laser systems with compensation for intracavity phase inhomogeneities (see, e.g., [2]). It also attracts attention as a system with rich temporal dynamics [3–6], from regular to chaotic [3], including cosine modulation of the output intensity and nearly triangular periodic pulses with deep ($0-\pi-0-\pi\ldots$) phase modulation [6].

The schematic representation of the considered oscillator is shown in Fig. 1. The cavity is formed by a conventional mirror M_c and a four-wave-mixing phase conjugate mirror in a photorefractive crystal (PRC). The crystal is pumped with two counterpropagating waves, 1 and 2, generating for any incident wave, 4, a backpropagating wave, 3. With a properly selected coupling constant γ, crystal thickness l, and pump intensity ratio $r=I_2/I_1$, the amplified reflectivity of a phase conjugate mirror compensates for all cavity losses and the waves 3 and 4 self-develop spontaneously; i.e., the oscillation occurs.

It is known that the reflectivity $|A_3^*(l)/A_4(l)|^2$ of a passive four-wave-mixing mirror increases in case of nearly degenerate four-wave mixing [7] as well as when the two pump waves are slightly misaligned [8]. Intuitively, one could expect that this will lead to a decrease of the oscillation threshold and an enhancement of the oscillation output. This is not always true, however, because both the pump misalignment and the frequency shift of the signal wave result in the appearance of an additional nonlinear phase in the reflected wave (the amplitude phase conjugate reflectivity $\rho=A_3^*/A_4$ becomes complex). Such an additional phase needs to be compensated to ensure the self-reproduction of the oscillation wave field after each complete double round trip of the cavity. One option to meet the complex threshold condition consists in ensuring the additional phase, which is equal to 2π [9]. This is possible, however, only for a quite large coupling strength, $\gamma l \geqslant 2\pi$.

The other way consists in compensating for undesirable phase shifts by using the frequency-shifted feedback [10], for example, with the piezomounted conventional mirror that introduces a frequency shift Ω_M to the wave that is reflected back into the cavity. It proved to be rather efficient in the experiments with a semilinear oscillator with a $BaTiO_3$:Co crystal, showing stable oscillation with no intensity modulation within a wide range of feedback frequencies Ω_M [10].

In this Letter we describe the behavior of a semilinear oscillator that is subjected to both a simultaneous feedback frequency shift and a pump misalignment. The numerical simulations and experimental results show that a combined action of these two factors can decrease the oscillation threshold and enhance an oscillation with only one frequency in each

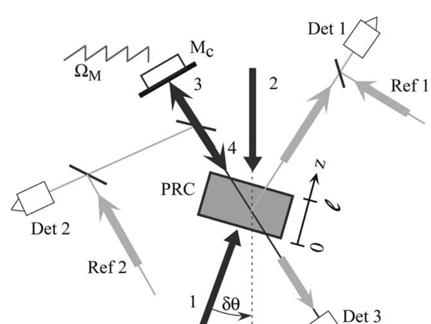

Fig. 1. Schematic representation of the semilinear oscillator with the waves 1 and 2 pumping the photorefractive crystal (PRC) and oscillation waves 3 and 4. A conventional cavity mirror M_c is mounted on a piezoceramic. Det 1, Det 2, Det 3 are the detectors; Ref 1 and Ref 2 are the reference waves for homodyne detection.

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wave, 3 and 4 (beat-free output). They also reveal parameter domains with a multiline oscillation spectrum.

Following [11] the equations that describe the time evolution of the system are formulated as

$$\frac{\partial A_1}{\partial z} = v^* \exp\left(i\frac{\Delta z}{2}\right)A_4, \quad (1)$$

$$\frac{\partial A_2}{\partial z} = v \exp\left(i\frac{\Delta z}{2}\right)A_3, \quad (2)$$

$$\frac{\partial A_3}{\partial z} = v^* \exp\left(-i\frac{\Delta z}{2}\right)A_2, \quad (3)$$

$$\frac{\partial A_4}{\partial z} = v \exp\left(-i\frac{\Delta z}{2}\right)A_1, \quad (4)$$

$$\tau\frac{\partial v}{\partial t} + v = \frac{\gamma^*}{I_0}\left[A_1^*A_4 \exp\left(i\frac{\Delta z}{2}\right) + A_2A_3^* \exp\left(-i\frac{\Delta z}{2}\right)\right], \quad (5)$$

where z is the coordinate along the propagation axis, $\mathbf{\Delta}=\mathbf{k_1}+\mathbf{k_2}-\mathbf{k_3}-\mathbf{k_4}$ is a wave-vector mismatch that appears because of misalignments of pump waves 1 and 2 to an angle $\delta\theta$ (see Fig. 1), A_i and $\mathbf{k_i}$ are the complex amplitudes and wave vectors of the waves $i=1,2,3,4$, A_i^* are their complex conjugates, γ is the coupling constant of the photorefractive crystal, v is the grating amplitude, I_0 is the total intensity $I_0 =|A_1|^2+|A_2|^2+|A_3|^2+|A_4|^2$, and τ is the response time of the photorefractive medium.

The modified boundary condition for the considered semilinear oscillator takes into account the frequency shift Ω_M introduced by a piezomirror [10],

$$A_4(z=l,t) = \sqrt{R}A_3(z=l,t)\exp(i\Omega_M t), \quad (6)$$

where R is the reflectivity of the conventional mirror.

For any set of dimensionless parameters γl, r, R, $\Omega_M\tau$, and Δl the dynamics of the oscillation intensity $|A_3|^2/|A_2|^2$ is calculated, and two quantities, the mean intensity and the beat frequency, are extracted from the steady-state domain. The simulations are repeated for different $\Omega_M\tau$ and Δl while keeping the same γl, r, R. The results are presented as 2D contour plots of the oscillation intensity in Fig. 2 for $\gamma l =4.8$, $r=2$, and $R=0.3$. The gray color in Fig. 2 marks the areas in which the intensity modulation occurs. One can see in Fig. 2 two pronounced regions with high intensity, with the maxima moved out of $\Omega_M\tau =0$ and $\Delta l=0$. These two mussel-shaped "mountains" correspond to beat-free oscillation, with single frequency waves 3 and 4.

Similar simulations for the other reasonable sets of parameters γl, r, R that can be reached experimentally confirm the above conclusion: the position of the maxima changes but they are never located on the lines that correspond to $\Omega_M\tau=0$ or $\Delta l=0$. Thus, the first important conclusion of the simulations is that

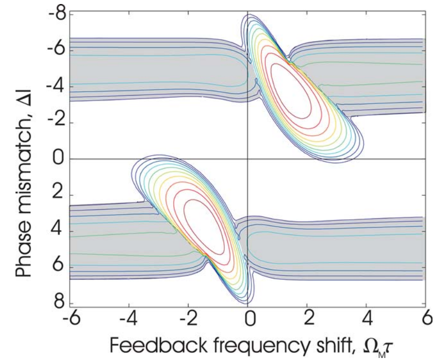

Fig. 2. (Color online) Contour plots of the oscillation intensity as a function of a frequency shift $\Omega_M\tau$ introduced by the vibrating cavity mirror and a phase mismatch Δl. All curves are separated by 20% of the maximum intensity between each. The areas with two frequency components in every oscillation wave, 3 and 4, are filled with gray color.

the common action of pump misalignment and frequency-shifted feedback may lead to an enhancement of the oscillation, provided that the signs and values of $\Omega_M\tau$ and Δl are chosen correctly.

The obtained solutions (Figs. 2 and 3) are invariant with respect to the simultaneous change of $\Omega_M\tau \to -\Omega_M\tau$ and $\Delta l \to -\Delta l$. The data of Fig. 2 show that, within certain intervals of the pump tilt angle, the nondegenerate oscillation exists and becomes insensitive to the mirror vibrations for sufficiently high $\Omega_M\tau$. These areas correspond to the excitation of the mirrorless oscillation, with the threshold reduced below $\gamma l=4.8$ because of the pump misalignment.

The experiments are performed with a new photorefractive crystal, $KNbO_3$ double-doped with Fe and Ag [12], which served as a gain medium. Its advantage as compared to $BaTiO_3$:Co is in its enhanced response at high spatial frequencies, which is important for the reflection grating geometry of the coherent oscillator used in the present experiment. With loosely focused pump waves of 200 mW power the photorefractive decay time was about 50 ms.

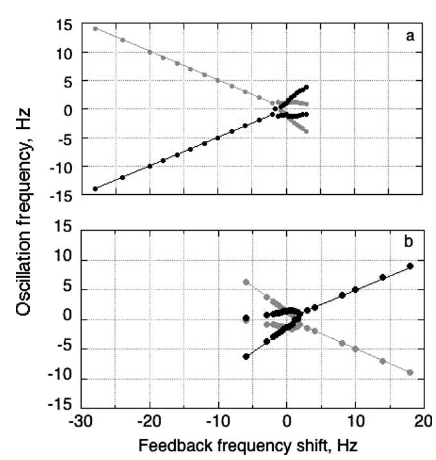

Fig. 3. Measured dependences of oscillation spectra on frequency shift Ω_M introduced by the vibrating cavity mirror. Graphs a and b are plotted for nearly symmetric positive and negative misalignment of the pump waves, $\Delta l \simeq \pm 1.5$. The spectra of waves 3 and 4 are shown by gray and black colors, respectively.

The $KNbO_3$ sample is pumped with a 514 nm Ar^+ laser (TEM_{00}, multiple longitudinal modes, ≃200 mW output power). Two pump waves impinge upon the sample with 4 mm×8 mm×7 mm dimensions through z faces at 25° with respect to the crystal z axis, while the cavity axis makes 20° with this axis. The measured coupling strength for two counterpropagating waves is $\gamma l \approx 4.2$.

The oscillation waves 4 and 3 are sent to the detectors Det 1 and Det 2 (Det 1 collects a part of wave 4 reflected from the sample face). The coherent reference waves Ref 1 and Ref 2 (with the pump waves frequency) are also directed to these detectors, which allows the measurement of the individual spectra of waves 3 and 4. The detector, Det 3, measures the intensity of wave 4 transmitted through the sample.

Figures 3 and 4 show typical examples of feedback frequency dependences of the oscillation spectrum and oscillation intensity, respectively, for a pump ratio $r=2$. The upper and lower graphs in each figure correspond to roughly symmetric positive and negative pump misalignment $\delta\theta \approx \pm 0.5$ mrad (i.e., with $\Delta l \simeq \pm 1.5$). It is obvious that the dependences for positive and negative misalignments are nearly symmetric with respect to a zero feedback frequency, $\Omega_M=0$. In the vicinity of $\Omega_M=0$ there are two different frequencies in each oscillation wave. With the feedback frequency beyond $|\Omega_M| \geqslant 7$ the oscillation frequency either does not exist or has only one component in spectrum. With diminishing misalignments, the regions with split oscillation frequencies disappear; this is in agreement with the contour plot shown in Fig. 2. As it follows from Fig. 4, the largest oscillation intensity is reached in the regions where the oscillation spectrum is not split.

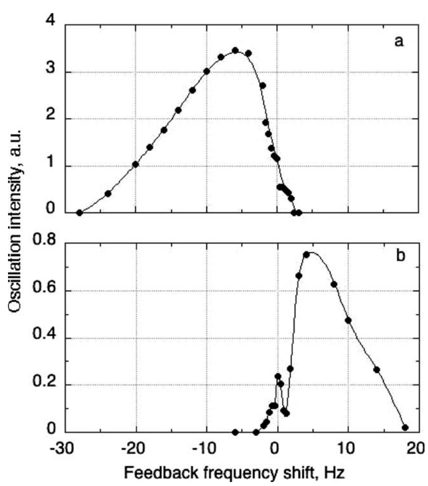

Fig. 4. Measured dependences of oscillation intensity (Det 3 in Fig. 1) on frequency shift Ω_M introduced by the vibrating cavity mirror. Graphs a and b are plotted for nearly symmetric positive and negative misalignment of the pump waves, $\Delta l \simeq \pm 1.5$. The same experimental conditions as used for Fig. 3.

The maxima of the oscillation intensity in Fig. 4 are located at $\Omega_M \approx \pm 5$ Hz. This is in reasonable agreement with the results of the simulation presented in Fig. 2: there the maxima are expected at $\Omega_M \tau \approx \pm 1.5$ when $\tau \approx 0.05$ s and $\Omega_M \approx \pm 5$ Hz. The calculated phase mismatch $\Delta l \simeq \pm 3.6$ that could ensure the observed scenario with a nondegenerate oscillation in the vicinity of $\Omega_M=0$ followed by beat-free oscillation when feedback frequency increases is more than two times different from that evaluated experimentally, $\Delta l \simeq \pm 1.5$. Thus only a qualitative agreement of the experimental data with the results of the simulation can be stated.

The experiment also revealed feedback frequency dependences that are more complicated compared to those shown in Fig. 3. For example, the beat-free domain may exist between two domains with nondegenerate oscillation. The detailed description of all the varieties of the experimental data and their interpretation will be given elsewhere.

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