

Semilinear coherent optical oscillator with frequency shifted feedback

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Abstract: It is shown that the saw-tooth variation of the cavity length in a photorefractive semilinear coherent oscillator can suppress the instability in the frequency domain and prevent a bifurcation in the oscillation spectrum. To achieve such a suppression the frequency of the cavity length modulation should be chosen appropriately. It depends on the photorefractive crystal parameters (electrooptic properties, photoconductivity, dimensions) and on the experimental conditions (pump intensity ratio, orientation of the pump and oscillation waves with respect to the crystallographic axes, polarization of the pump waves, etc.). It depends also strongly on a possible misalignment of the two pump waves. On the other hand, within a certain range of the experimental parameters the mirror vibration may lead to a further frequency splitting in the already existing two-mode oscillation spectrum.

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OCIS codes: (190.4380) Nonlinear optics, four-wave mixing; (190.5330) Photorefractive optics; (230.4910) Oscillators.

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1. Introduction

A frequency shifted feedback in cavities of conventional lasers is known from 1965 when Foster et al. [1] demonstrated mode locking of He-Ne laser by adjusting the feedback frequency shift to one longitudinal mode spacing. Since that time more than hundred publications appeared on this topic, theoretical as well as experimental. The reader can find a list of the most important references in the recent publications [2,3]. Quite often the main objective of frequency shifted feedback in conventional lasers is to spread a coherent light spectrum over a broad range of frequencies [2]. In what follows it will be proved that a frequency shifted feedback can fulfill an opposite task in coherent oscillators operating with four-wave mixing gain (see, e.g., [4]) : the two-frequency output can be converted into a single-frequency output. This becomes possible in coherent oscillators with a four-wave mixing phase conjugate mirror which serves as an adaptive element. Besides that, with the appropriate choice of the parameters of the system, the frequency shifted feedback may result also in the appearance of a more rich oscillation spectrum as compared to the oscillator with the immobile conventional mirror.

As distinct from the conventional lasers where the frequency shift of the feedback is comparable to the longitudinal cavity mode spacing, in photorefractive oscillators the frequency shift should be comparable to the index grating decay rate, i.e., it should be many orders of magnitude smaller than the longitudinal mode spacing.

The structure of this article is as follows: First, the particular features of a semilinear coherent oscillator with two counterpropagating pump waves are reminded and the motivation of the proposed modification that involves the frequency shifted feedback is given. Then the experimental results are presented and the conclusions of the linear stability analysis are discussed. Finally, the multifrequency oscillation is qualitatively explained.

2. Semilinear coherent oscillator with vibrating mirror

A semilinear coherent oscillator with two counter propagating pump waves (Fig. 1) consists of a conventional mirror M and a phase conjugate mirror PCM based on four-wave mixing in a photorefractive crystal. Two counterpropagating coherent pump waves (1,2) with specially selected polarization and intensity ratio impinge upon the photorefractive crystal to ensure, for any seed wave 3, the appearance of the phase conjugate wave 4 with a higher intensity. Due to the adaptive properties of the phase conjugate mirror the oscillation in such a device is insensitive to the cavity length and most often its frequency is exactly equal to the frequency of the pump waves.

Several years ago it was discovered however that the frequency degenerate coherent oscillation in an oscillator like that shown in Fig. 1 may become unstable for a sufficiently high coupling strength [5,6]. Within a certain domain of pump intensity ratios a single frequency in the oscillation spectrum splits into two modes shifted symmetrically with respect to the frequency of the pump waves. The splitting is observed with the decrease of the pump intensity ratio from large values (strong disbalance) to unity (equal intensities of the two pumps); it occurs via a supercritical bifurcation [6]. The appearance of two modes in the spectrum results in high contrast temporal variations of the oscillation output intensity what is not always desirable. We show in this paper that by introducing a feedback frequency shift via an appropriate linear modulation of the reflected wave phase (with the help, e.g., of an electro optic modulator

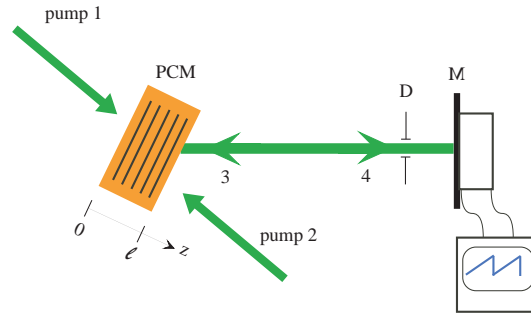


Fig. 1. Schematic representation of a semilinear coherent oscillator. Two counterpropagating pump waves (1,2) with the frequency ω_{pump} impinge upon the sample. The oscillation occurs between the photorefractive crystal that serves as an amplifying phase conjugate mirror (PCM) and the conventional mirror M mounted on a piezoceramic holder. An aperture D is placed inside the cavity for the transverse mode control.

or a piezomounted conventional mirror) it is possible to transform a two-frequency oscillation into a single mode oscillation with its frequency shifted with respect to that of the pump wave.

To explain this behavior it is useful to consider the self-reproducibility of the lasing wave frequency after a complete round trip in the cavity with the diagrams drawn in Fig. 2. The laser cavity is shown, with the phase conjugate mirror PCM (yellow) together with the conventional mirror that can be immobile (mirror M in gray) or can move (vibrating mirror VM in blue). While the abscissa of any frame of this figure gives a real coordinate along the cavity optical axis, the ordinate shows the temporal frequency of a particular wave component. The red arrows indicate the direction of propagation of the oscillation wave components.

It is important to emphasize that the two mirrors of the cavity transform the temporal frequency of an incident wave in a quite different manner: The phase conjugate mirror changes the sign of the frequency detuning with respect to the pump wave to the opposite, the ordinary mirror with the saw-tooth variation of the reflecting surface position shifts the frequency up or down by a constant value that does not depend on the frequency of the incident wave. The lines inside the boxes that define the mirrors in Fig. 2 have no physical meaning, they only show how the frequencies are transformed during the reflections.

To start with, we consider the frequency degenerate oscillation in a cavity with the immobile conventional mirror (Fig. 2(a)). Here the oscillation wave keeps its frequency (which is equal to the frequency of the pump waves) when being reflected from the conventional mirror and also from the phase conjugate mirror. It should be noted that even in the degenerate case two round trips are necessary to reproduce the oscillation wave field, as in any cavity with a phase conjugate mirror [7].

In the same set-up with an immobile conventional mirror the degeneracy is broken for high coupling strengths at small pump ratios [5,6], this is illustrated in Fig. 2(b). The phase conjugate mirror transforms the wave 4 into the phase conjugate wave 3 and changes the frequency $\omega_4 = \omega_{pump} + \Omega$ to the frequency $\omega_3 = \omega_{pump} - \Omega$. Similarly, the wave 4' with frequency $\omega_4' = \omega_{pump} - \Omega$ is transformed into wave 3' with frequency $\omega_3' = \omega_{pump} + \Omega$. The immobile conventional mirror does not affect the frequency of the reflected waves, $\omega_4 = \omega_4'$ and $\omega_3 = \omega_3'$. It is evident that the oscillation wave field is reproduced after two consecutive round-trips in the cavity, with the sequence of wave transformation $4 \rightarrow 3 \rightarrow 4' \rightarrow 3' \rightarrow 4$. Two oscillation components with different frequencies propagate in both directions, from the conventional mirror to the phase conjugate one and vice versa. The interference of these components results in a deep modulation of the output oscillation intensity [5].

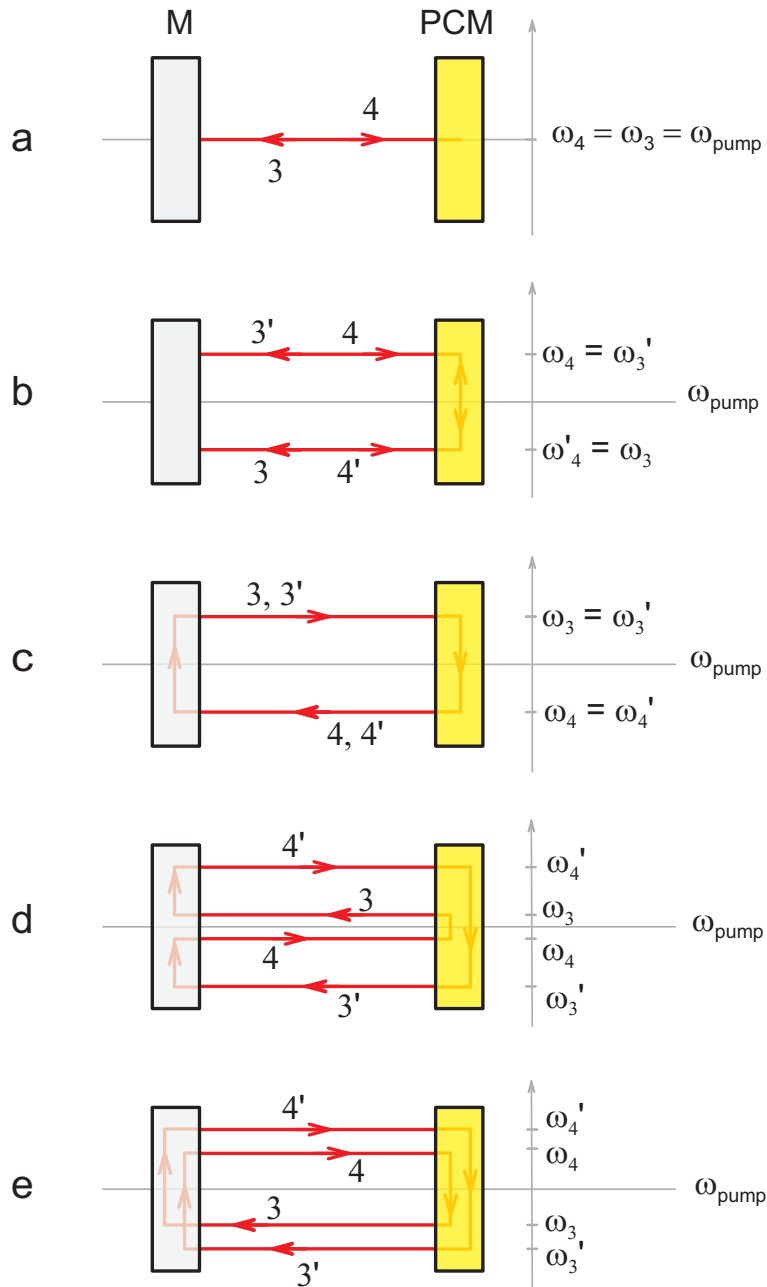


Fig. 2. Diagrams of oscillation frequency self-reproduction in the cavity formed by a phase conjugate mirror (PCM) and a conventional mirror that can be immobile (M) or can move (vibrating mirror VM). The central horizontal line in each frame marks the temporal frequency of the pump wave ω_{pump} . The displacement up and down from this line marks positive and negative detuning with respect to the pump frequency, respectively. The numbers inside the cavity (3, 3', 4, 4') label the particular components of the oscillation mode, the arrows show the direction of propagation. The frames (a,b) depict the cavity with the immobile conventional mirror while frames (c,d,e) depict those with the saw-tooth modulation of the mirror position.

To avoid this intensity modulation, the intuitive solution is to introduce a feedback frequency shift Ω_M (subscribe M indicates that the frequency shift is introduced by the vibrating mirror VM) that is equal exactly to 2Ω , as shown in Fig. 2(c). By this means, the wave 4 with frequency $\omega_4 = \omega_{pump} - \Omega$ is transformed by the vibrating mirror into wave 3 with frequency $\omega_3 = \omega_{pump} + \Omega$. Once more, two round trips of the cavity are needed to self-reproduce the oscillation wave, with the similar sequence of the wave transformations, $4 \rightarrow 3 \rightarrow 4' \rightarrow 3' \rightarrow 4$. At the same time the wave with only one temporal frequency propagates in any direction inside the cavity and the output intensity is not modulated. This is substantially different from the case depicted in Fig. 2(b).

This idea was confirmed by the experiments. Moreover the analysis of the set of equations that describes the dynamics of the considered oscillator yields reasonably low thresholds for such an oscillation. Figure 2 contains two more diagrams that show other possible conditions of frequency self-reproduction for the oscillator with a frequency shifted feedback. We will return to the discussion of these cases after the description of the experimental results.

3. Experiment

The oscillator consists of a BaTiO₃:Co crystal pumped by a TEM₀₀ Ar⁺-laser emitting at 514 nm (no etalon inside the cavity) and a concave mirror with focal length $F = 25$ cm. By a special selection of the path difference of the pump and oscillation waves the creation of the reflection photorefractive grating is favoured [8]. The cavity length is 25 cm and a 0.7-mm aperture is placed inside the cavity. The geometry of the interaction is optimized to ensure high two-beam coupling gain as it was described in [9]. A beam splitter is inserted inside the cavity to extract a part of the oscillating waves 3 and 4. Each of these waves interfere with a reference beam formed from the pump beam 2. In case of a non degenerate oscillation, the interference fringes motion is recorded with a detector whose aperture is smaller than the fringe spacing. The analysis of the temporal frequency content in the signal is performed by taking the Fourier transform. All other details reproduce the experimental conditions of ref.8.

The new feature as compared to [8] consists in the possibility to vibrate the conventional mirror of the cavity which is mounted on a piezoceramic holder. The saw-tooth voltage from a signal generator is applied to the piezoceramic. The maximum voltage is chosen such that it ensures a mirror displacement of one half wavelength of the pump wave. This corresponds to a 2π phase variation of the wave reflected by the mirror M. In such a way a frequency shift is introduced in the reflected wave, which is equal to the frequency of the saw-tooth modulation. The sign of the frequency shift depends on the sign of the applied voltage; with a positive voltage the mirror moving shortens the cavity length so that the frequency shift is positive.

First, let us consider the oscillation in the cavity with the immobile mirror, where the spectrum contains only one frequency for a given set of pump ratio $r = I_2/I_1$, coupling strength $\gamma\ell$, and conventional mirror reflectivity R (i.e., the case of Fig. 2(a)). If the conventional mirror starts to vibrate and introduces a frequency shift Ω_M the frequency of the oscillation wave 3 moves up by $\Omega_M/2$ while that of the wave 4 moves down by $\Omega_M/2$, i.e. the oscillator operates according to the case of Fig. 2(c). The linear relationship of frequencies holds throughout a whole range of the existence of the oscillation (Fig. 3).

In the next step, the mirror M being immobile, we reduce the pump intensity ratio so that two modes appear, shifted symmetrically by $\pm\Omega$ with respect to the pump frequency ω_{pump} . Then a saw tooth voltage with frequency $|\Omega_M|$ is applied to the piezoceramic so that a frequency shift is induced on the oscillation waves resulting in the beat frequency $\omega_4 - \omega'_4 = \omega_3 - \omega'_3$ in the oscillation wave intensity. This beat diminishes and finally becomes zero as the feedback frequency shift Ω_M is increased up to $\Omega_M = 2\Omega$. For a further increase of $|\Omega_M|$ no beats are observed. Thus, the appropriate choice of the feedback frequency shift removes the temporal

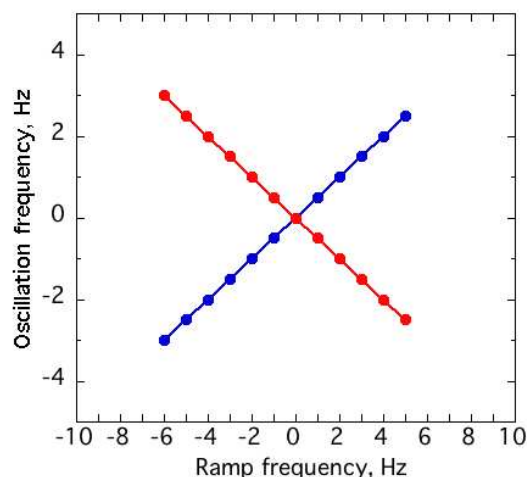


Fig. 3. Oscillation frequency versus frequency detuning introduced by a piezo-mirror. The pump intensity ratio is $r = 120$. The data related with the oscillation waves 3 and 4 are in red and blue, respectively.

modulation of the oscillation intensity.

The range of Ω_M with the multifrequency oscillation and the type of response of the oscillation spectrum to the frequency shift of the feedback is very sensitive to a slight misalignment of the two counterpropagating pump waves. A tilt of the wave 1 of only 10^{-4} rad transforms the dependence shown in Fig. 4 into that shown in Fig. 5. In this latter situation, the beat frequency increases with increasing $|\Omega_M|$ and suddenly disappears beyond a certain range of Ω_M (see Fig. 5). It is important to note that the cw output with no modulation is always achieved at sufficiently large feedback frequencies, in spite of qualitatively different behavior at small feedback frequencies.

The data of Fig. 4 and Fig. 5 show that within the range of small feedback frequency shifts the oscillation mode may contain four different frequency components. The relevant diagrams of frequency self-reproduction are shown in Fig. 2(d-e). As distinct from the case of Fig. 2(c), here the degeneracy of the waves 3 and 3' (as also of the waves 4 and 4') is removed. The two coherent components with different frequencies propagate in each direction inside the cavity (3, 3' and 4, 4'). Thus, the intensity modulation occurs and the frequencies of the waves 3 and 4 in the general case have nothing to do with the feedback frequency shift Ω_M , they may be either larger or smaller than Ω_M , for the cases of Fig. 2(d-e), respectively. However, the frequencies $\omega_3 = -\omega_4$ may collapse to zero occasionally, as it happened at $\Omega_M \approx -2$ Hz in Fig. 4. In this case $\omega'_4 = -\omega'_3$ becomes equal to Ω_M . The wave 4 moves down i.e., the oscillator operates according to the case of Fig. 2(c) and the linear relationship between the oscillation frequency and the ramp frequency is valid (Fig. 3).

4. Stability analysis

The dynamics of the oscillation in the considered coherent oscillator is described by the set of propagation equations for the four complex amplitudes of interacting waves A_i , $i = 1, 2, 3, 4$, and an equation for the temporal variation of the index grating amplitude v (see, e.g. [10]). To analyze the threshold conditions it is sufficient to consider the undepleted pump approximation,

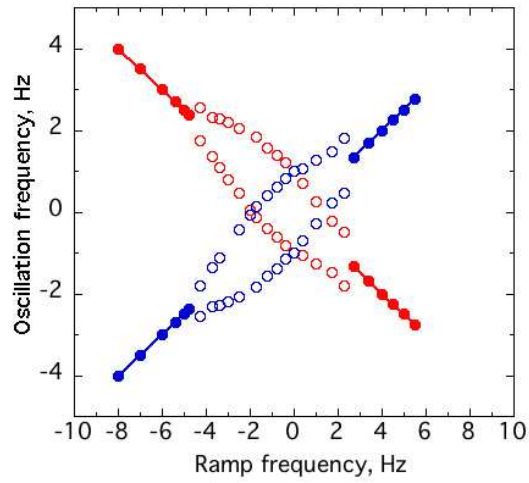


Fig. 4. Oscillation frequency versus frequency detuning introduced by a piezo-mirror. The pump intensity ratio is $r = 2.7$. The data related with the oscillation waves 3 and 4 are in red and blue, respectively.

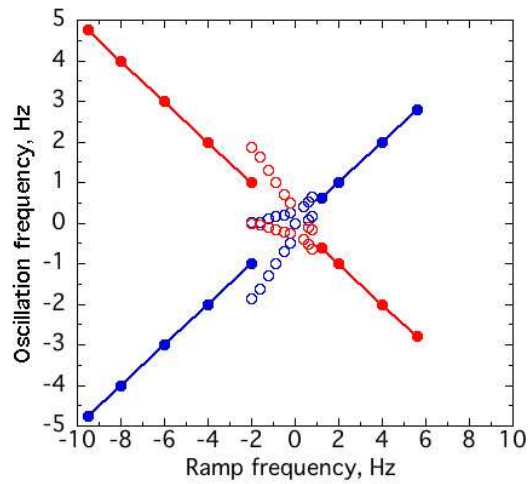


Fig. 5. Oscillation frequency versus frequency detuning introduced by a piezo-mirror. The pump ratio is $r = 2.7$. The data related with the oscillation waves 3 and 4 are in red and blue, respectively. The pump waves are tilted for 0.1 mrad with respect to the alignment of previous figure.

$\partial A_1/\partial z = \partial A_2/\partial z = 0$, i.e., only three equations are necessary:

$$\frac{\partial A_3}{\partial z} = v^* A_2^\ell, \quad (1)$$

$$\frac{\partial A_4}{\partial z} = v A_1^0, \quad (2)$$

$$\tau \frac{\partial v}{\partial t} + v = \frac{\gamma}{I_0} (A_1^{0*} A_4 + A_2^\ell A_3^*). \quad (3)$$

where τ is a constant response time, $A_2^\ell = A_2(z = \ell, t) = \text{const}$, $A_1^0 = A_1(z = 0, t) = \text{const}$, ℓ is the crystal thickness and I_0 is the total intensity : $I_0 = I_1^0 + I_2^\ell = \text{const}$. The asterisk stands for the complex conjugation. The difference from the oscillator with the immobile mirror is in the boundary condition

$$\begin{aligned} A_4(\ell, t) &= \sqrt{R} A_3(\ell, t) \exp(i\Omega_M t), \\ A_3(0, t) &= 0. \end{aligned} \quad (4)$$

The first above boundary condition takes into account the feedback frequency shift Ω_M imposed by the vibrating mirror. An ansatz relevant with the equations (1-3) and the boundary conditions (4) has the form :

$$\begin{aligned} A_3(z, t) &= a_3(z) \exp[(p - i\Omega_M/2)t], \\ A_4(z, t) &= a_4(z) \exp[(p + i\Omega_M/2)t], \\ v(z, t) &= n(z) \exp[(p + i\Omega_M/2)t]. \end{aligned} \quad (5)$$

Here p is an exponent and $\Omega_M/2$ is the frequency detuning of the oscillation wave with respect to the pump waves.

By substituting Eqs. 5 into Eqs. 1-3 and taking into account the boundary conditions (Eqs. 4), one arrives to the equation for the exponent p

$$Rr = \frac{\exp(2a) + 2r \exp(a) \cos b + r^2}{\exp(2a) - 2 \exp(a) \cos b + 1}, \quad (6)$$

with

$$\begin{aligned} a &= \frac{\gamma^\ell (1 + \tau p)}{(1 + \tau p)^2 + (\Omega_M \tau/2)^2}, \\ b &= \frac{\gamma^\ell \Omega_M \tau/2}{(1 + \tau p)^2 + (\Omega_M \tau/2)^2}. \end{aligned} \quad (7)$$

The details of the linear stability analysis will be presented elsewhere, here we outline only the main results important for the discussion:

(i) The instability according to the diagram of Fig. 2(c), with a frequency detuning $+\Omega$ or $-\Omega$ which is equal to $\Omega_M/2$ appears in a wide range of coupling strengths and pump intensity ratios.

(ii) The solutions of Eq. 6 are symmetric with respect to the frequency detuning : the change of $+\Omega_M$ to $-\Omega_M$ results in no change of Eq. 6.

(iii) The solutions of Eq. 6 are insensitive to a simultaneous change of the sign of coupling strength, $\gamma^\ell \rightarrow -\gamma^\ell$ and reversion of the pump intensity ratio, $r \rightarrow 1/r$. This known property of the conventional semilinear coherent oscillator [11] remains valid also in the case of oscillator with frequency shifted feedback.

It is important to add that Eq. 6 can be reduced to a simple condition of oscillation, $R_{pc}R = 1$, with the standard expression for the phase conjugate reflectivity [11]

$$R_{pc} = \frac{\sinh^2(a/2) + \sin^2(b/2)}{\cosh^2[(a/2) - (\ln r)/2] - \sin^2(b/2)}. \quad (8)$$

This means that the phase condition of the oscillation is met for any possible feedback frequency detuning automatically for our oscillator, providing the condition $|\Omega| = |\Omega_M/2|$ holds.

Exactly at the threshold of oscillation $p = 0$ and Eq. 6 allows to define the threshold coupling strength for any pre-imposed frequency shift $\Omega_M = 2\Omega$. A representative example is shown in Fig. 6 for a low-loss cavity ($R = 1$) and the pump ratio $r = 1.5$. We remark that for small Ω_M the threshold coupling strength at first decreases with increasing detuning; the smallest threshold $\gamma_{th}^{\ell} \approx 2.49$ is reached at $\Omega_M = \tau/2$. The important conclusion that can be made from this

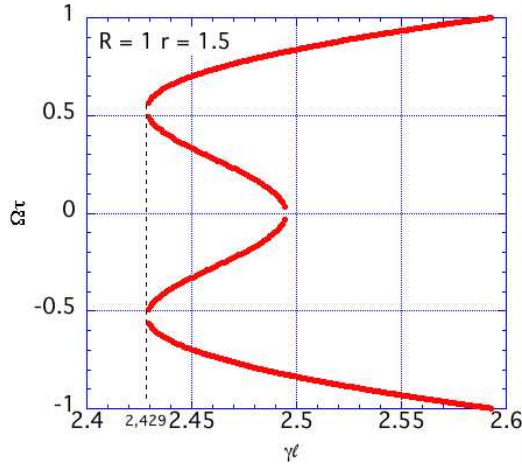


Fig. 6. Calculated threshold coupling strength versus frequency detuning of the oscillation which is controlled by the feedback frequency shift Ω_M for $R = 1$ and $r = 1.5$.

graph is that the shifted frequency oscillation can be reached at moderate values of coupling strength. This is different from the prediction of natural splitting into two lines for spectrum of oscillator with the immobile mirror [10]. In this latter case, the stability analysis was conducted with another ansatz so that another class of solutions was found with different characteristics: in particular, the splitting into two lines for the spectrum is expected to occur only above $\gamma_{th}^{\ell} = 2\pi$. It follows from Fig. 6 that, for a given pump ratio, there is a range of frequency detunings for which the oscillator has a lower threshold in comparison with that where there is no frequency detuning. This should result in a higher output intensity for a shifted frequency (whatever is the sign of the detuning), what was really observed with the experimental conditions of Fig. 4.

Thus the results of the stability analysis are in agreement with the data of Fig. 3. They are also in agreement with the experimental data of Fig. 4 and Fig. 5 for a sufficiently large frequency detuning introduced by the vibrating mirror, i.e., in a range of Ω_M where the frequency degeneracy is removed. The stability analysis of a multi frequency oscillation at a moderate coupling strength needs a more complicated model; it will be necessary, probably, to go beyond the plane wave approximation or to consider a slight deviation from the exact phase matching.

5. Conclusion

To conclude, we described a new modification of the semilinear photorefractive coherent oscillator with a frequency shifted feedback. By this mean, it is shown that an oscillation free from intensity modulation can be achieved even at high values of the coupling strength and small values of pump ratio, i.e., in the range where usually a two-frequency oscillation occurs.

The linear stability analysis shows that such a type of oscillation is possible with moderate values of the crystal coupling strength.

The experiment confirms qualitatively well our expectations: In the oscillator with no natural frequency split (i.e., in oscillator with rather large pump intensity ratio) we measure the oscillation frequency Ω which is exactly equal to $\Omega_M/2$. To suppress the instability in the spectral domain (in oscillator with rather small pump intensity ratio) the frequency modulation of the feedback should overpass a certain threshold frequency which falls in the interval between natural frequency shift at $\Omega_M = 0$ and its doubled value.

In the experiment we observe also more complicated operation modes, with four frequencies excited simultaneously by the moving mirror. Their appearance may be related to a slight misalignment of the counterpropagating pump waves; it may have the same origin as two-frequency oscillation at small coupling strengths, described in [4,5].

Acknowledgments

The authors thank B. Sturman for his help in the stability analysis. The authors are grateful to Daniel Rytz for the BaTiO₃:Co samples used in the experiments and to Gregory Gadret for his expert help in the experiment. Serguey Odoulov acknowledges Institut Carnot (Université de Bourgogne) for an invitation as visiting professor.