Light pulse amplification by photorefractive two-wave mixing

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Abstract: It has been shown recently (see, e.g., Nature, **397**, 594, 1999; **409**, 490, 2001) that light pulses can be remarkably slowed down in resonant gases under very low temperatures. We show theoretically and experimentally that a similar in nature (and stronger in value) phenomenon occurs in photorefractive nonlinear media. The pulse-maximum velocity can be reduced here to the value of *thickness/response time* because of the two wave coupling effect. Specific properties of pulse propagation depend on the type of the photorefractive response and also duration and shape of the input pulse. Our theoretical results for the nonlocal nonlinear response are in a good agreement with experimental data obtained with $BaTiO_3$ and $Sn_2P_2S_6$ crystals.

OCIS codes: (190.5330) Photorefractive nonlinear optics; (190.7070) Two-wave mixing.

Introduction

Our interest to pulse propagation in photorefractive (PR) media stems from resent results on slowing down of light pulses in resonant gases under very low temperatures [1,2] caused by the effect of electro-magnetically induced transparency (EIT) [3]. We argue that an effect similar in nature (but much stronger in value) occurs in photorefractive media under conditions usual for continuous-wave (CW) two-wave-mixing experiments. The pulse-maximum velocity of the pulse can be made here incomparably smaller than the speed of light. The similarity between the PR and EIT cases concerns the form of the initial nonlinear equations, the shape of output light signals, and the necessity to use input pulses whose width is larger than the nonlinear response time to avoid broadening effects. The nonlinear effects under study differ strongly from the linear effects of time dependent absorption [4].

Furthermore, to our best knowledge the elementary problem of pulse amplification by two-wave mixing was never considered in the literature for the CW-range. The known pulse experiments dealt mainly with grating recording and very short pulses, whose duration is comparable with (or smaller than) the characteristic microscopic times of the medium, see, e.g., [5-7]. The subject matter differs considerably from the known problems of transient beam coupling [8,9].

Schematic diagram of a light pulse amplification experiment is shown in Fig.1. Two coherent light beams are incident onto the same input face of a photorefractive crystal, $z = 0$, a pump beam of a constant amplitude and a signal pulse beam, whose input intensity peaks at $t = 0$. Our task is to find the time profile of the output beam and investigate the main tendencies of the nonlinear pulse propagation for two main types of the photorefractive response - the local and nonlocal response [10].

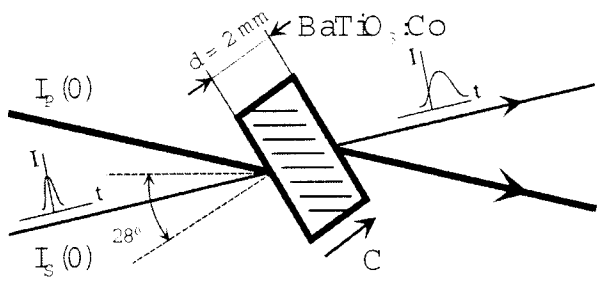

FIG. 1: Schematic of a pulse amplification experiment: *C* is the polar axis of the crystal.

The constituencies of the pulse amplification are not different from those typical of two-wave coupling. The pump and signal beams form a refractive index grating and experience Bragg diffraction from this grating. What makes our case specific is a restricted duration of the processes under study. If the pulse duration is much shorter than the PR response time, the grating recording is inefficient and the output signal intensity is not much different from the input one. The most interesting events are expected in the case when the input pulse duration is comparable with (or larger than) the response time. The leading edge of the input pulse cannot be strongly changed during propagation because of a weak effect of the light-induced index grating. As for the main body of the output pulse and (especially) its trailing edge, they can be affected strongly because of the recording inertia. A stronger amplification of the trailing edge results in delay of the pulse maximum and, therefore, in an effective slowing down of light propagation. The output shape of the pulse is expected to depend essentially on the crystal thickness and the type of the nonlinear response.

Theoretical background

Diffraction equations for the signal beam amplitude A and the pump beam amplitude A_p we write down in the conventional form [10,11],

$$\frac{\partial A}{\partial z} = -i\frac{\pi n^3 r}{\lambda} E A_p, \tag{1}$$

$$\frac{\partial A_p}{\partial z} = -i\frac{\pi n^3 r}{\lambda} E^* A . \tag{2}$$

Here $E = E(z,t)$ is the grating amplitude, n is the background refractive index, r is the relevant electro-optic coefficient, λ the wavelength, and the asterisk stands for complex conjugation. The light absorption is expected to be negligible.

The material equation for the grating amplitude E, which supplements the set (1) - (2), can be chosen as follows:

$$\left(\tau\frac{\partial}{\partial t}+1\right)E = E_s\frac{A A_p^*}{|A_p|^2+|A|^2}, \tag{3}$$

where τ is the response time (usually, it is inversely proportional to the total light intensity) and E_s the characteristic (generally complex) space-charge field which is known for many particular models of charge transport [10,11]. For the local and non-local responses E_s is real and imaginary, respectively.

Further, we shall use the undepleted pump approximation by assuming that $|A|^2 << |A_p|^2$. Within this approximation $A_p = const$, and Eqs. (1), (3) form a closed set of linear partial differential equations for A and E. Since the amplitude $A(z,t)$ tends to zero for $t \to \pm\infty$, one can use the Fourier transformation $A(z,t) \to A_\omega(z)$. After elementary calculations we have at the output, $A_\omega(d) = A_\omega(0)\exp[\gamma d (1 - i\omega\tau)^{-1}]$, where $\gamma = -i\pi n^3 r E_s/\lambda$ is the rate coefficient and $A_\omega(0)$ is the Fourier transform of the input amplitude $A(0,t)$. Generally, the rate coefficient is complex, $\gamma = \gamma' + i\gamma''$, the quantity $2\gamma'$ is the exponential intensity gain factor for two-wave coupling, which is zero for the local response.

To calculate the output intensity $|A(d,t)|^2$, one has to perform the inverse Fourier transform and take

the square of its absolute value. This can be done numerically for any particular shape of the input amplitude. From now on we restrict ourselves to the Gaussian input beam,

$$A(0,t) = A_0 \exp(-t^2 / t_0^2) \tag{4}$$

with A_0 being the input pulse amplitude and t_0 the input pulse half-width. Then $A(\omega,0) = \sqrt{\pi} A_0 t_0 \exp(-\omega^2 t_0^2 / 4)$ and for the output amplitude we obtain the following explicit relation:

$$A(d,t) = \frac{A_0 t_0}{2\sqrt{\pi}\,\tau} \int_{-\infty}^{\infty} \exp[-ix(t/\tau) - x^2 (t_0/2\tau)^2 + \gamma d (1-ix)^{-1}]\, dx . \tag{5}$$

The right-hand side is a function of the normalized time t/τ; its form is controlled by two dimensionless parameters, the ratio t_0/τ and the coupling strength γd.

Let us consider shortly the similarities between the PR and EIT cases. Both of them deal with nonlinear effects. The role of atomic coherence and its rise time play in our case the index grating and the PR response time, respectively. In the EIT case, the role of the rate coefficient $2\gamma'$ plays the light absorption coefficient. In the case of nonlocal response ($\gamma'' = 0$) there is one-to-one correspondence between the shapes of the output pulses if the EIT effect is driven far from saturation. The main distinction of the PR case is the presence of spatial amplification of the output pulse.

Theoretical and experimental results

We consider first the simplest case of the nonlocal response, when the rate coefficient γ is real. We are dealing here with the amplitude amplification while the phase distortions are absent. According to Eq. (5), the ratio $A(d,t)/A_0$ is real in this case.

Figure 2a shows the shape of the output pulse for $t_0/\tau = 1$ and three values of the coupling strength; the trailing edge is to the right of the maximum. The case $\gamma d = 0$ corresponds to free pulse propagation; the output intensity profile is the same here as the input profile. With the spatial amplification present, the output pulse is remarkably delayed against the input one; the larger γd, the longer is the delay time $\Delta t \approx \gamma d\,\tau$ (it is defined as the time of the output intensity maximum). The time delay is accompanied by the pulse amplification; the relevant scaling factors are indicated in Fig. 2a. The longer Δt, the larger is the peak value of $|A(d,t)|^2$ and the overall pulse power. Furthermore, the pulse width grows clearly with increasing γd.

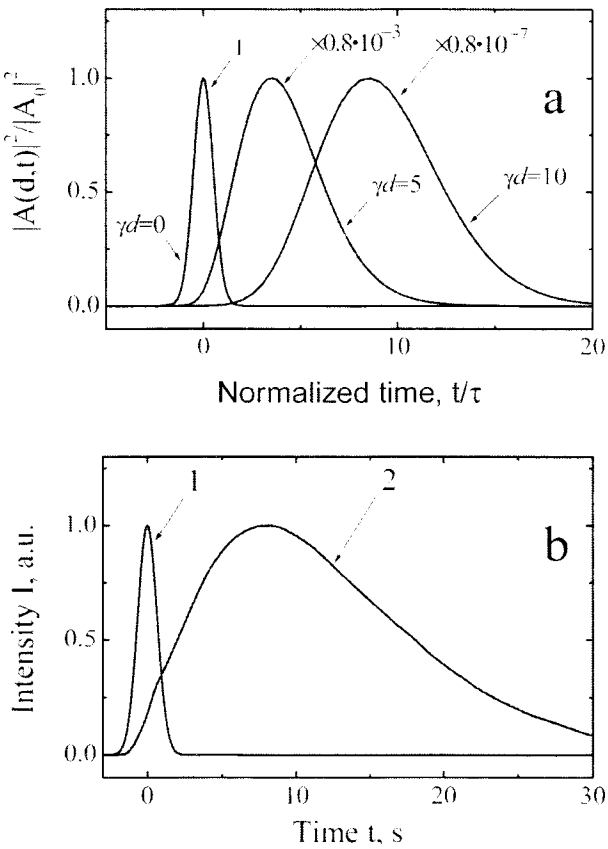

FIG. 2: (a) The normalized time dependences of the output intensity for the nonlocal response, $t_0/\tau = 1$; the lines 1,2,3 and the corresponding normalization factors are calculated for $\gamma d = 0$, 5, and 10, respectively. (b) Experimental dependences of the input (line 1) and output (line 2) intensities for $BaTiO_3$; the input half-width $t_0 = 0.92$ s.

Taking into account the output pulse delay the effective pulse velocity in the medium has to be evaluated as $d/\Delta t \approx 1/\gamma\tau$. Since the response time τ ranges from 10^{-2} to 10^{2} s in CW-experiments, this velocity is incomparably smaller than the speed of light. The underlying reason for this huge slowing down is indeed the specific inertial photorefractive two-wave mixing.

Most experimental data on pulse amplification we have obtained with a $BaTiO_3$:Co sample in an optical configuration shown in Fig. 1. The shape of the input pulse was PC-controlled via an electro-optic modulator; typically this shape was Gaussian. The main experimental parameters were as follows: The thickness $d = 2$ mm, the wavelength $\lambda = 633$ nm, the light absorption coefficient ≈ 1 cm^{-1}, the input pump intensity $\approx 3\,\mathrm{W/cm^2}$, the input intensity ratio $|A_p/A_o|^2 \approx 10^6$, the grating period $\approx 2.6\mu$ m, and the response time $\tau \approx 3$ s. No external electric fields were applied, hence the charge separation was due to the diffusion mechanism. The light beams were extraordinarily polarized; the coupling strength γd was estimated as ≈ 4.5 from auxiliary two-wave coupling experiments; this corresponds to $\gamma \approx 22\,\mathrm{cm^{-1}}$. Additionally, we used $Sn_2P_2S_6$ crystals showing a strong diffusion-type nonlocal response [12]. The response time of these crystals is typically much shorter than that of $BaTiO_3$.

Figure 2b shows representative experimental data obtained with our $BaTiO_3$ sample for $t_0 = 0.92$ s. One sees that the output pulse is substantially delayed and broadened, its shape is in a good qualitative agreement with the theory. The effective velocity of the pulse maximum can be estimated as ≈ 0.025 cm/s. Similar results have been obtained with $Sn_2P_2S_6$ crystals; the pulse velocity in these crystals was ≈ 40 cm/s. This is more than three orders of magnitude higher than the value of the speed typical of our $BaTiO_3$ sample. The scale of the light slowing down remains, nevertheless, very impressive.

It is remarkable that the described behavior occurs only for the values of the half-width t_0 that are sufficiently large as compared to the response time τ. In the opposite limit, new distinguishing features take

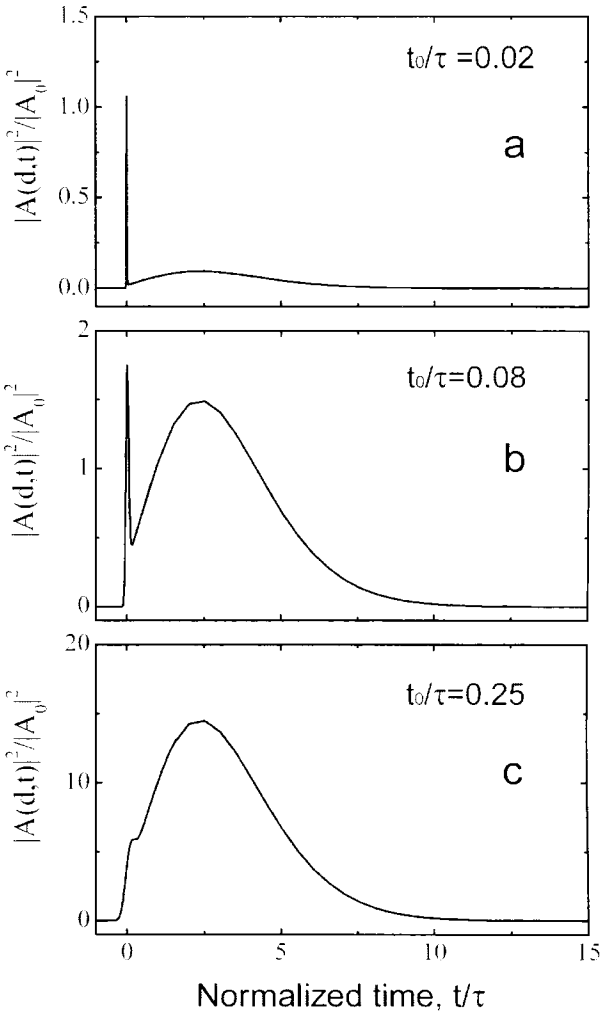

FIG. 3: The calculated shape of the output pulse for the nonlocal response, $\gamma d = 4$, and three values of t_0/τ.

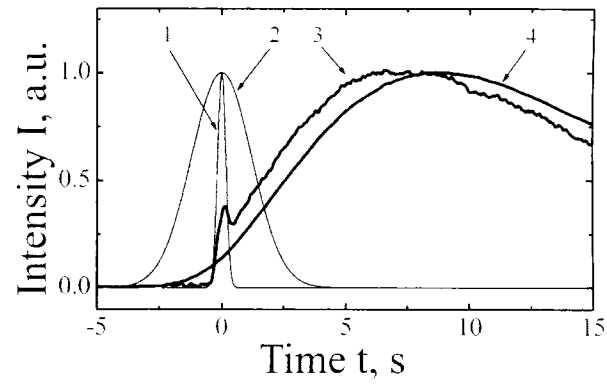

FIG. 4: Experimental dependences of the output intensity for $\gamma d \cong 4.5$ and two different values of t_0: the lines 3 and 4 correspond to $t_0 = 0.33$ and 2.4 s; the lines 1 and 2 show the input profiles.

place. The main tendencies of the pulse shape changes with increasing t_0/τ are shown in Fig. 3 for $\gamma d = 4$. For $t_0/\tau = 0.02$, the output pulse consists of a sharp practically non-shifted peak and a weak long tail. The height and width of the non-shifted peak are not much different from those of the input pulse. The tail generally possesses a maximum at $\Delta t >> t_0$. With increasing t_0/τ, the tail is growing in value, its maximum is gradually shifting to the right, whereas the non-shifted narrow peak is getting less and less pronounced. Lastly, for $t_0/\tau = 0.2$ the non-shifted peak has fully disappeared and what we can see at the output is a single delayed pulse, which is strong and wide. Note that the transition from the one-maximum to the two-maxima behavior occurs critically; for a certain chosen value of γd it corresponds to a unique threshold value of t_0/τ (this value equals ≈ 0.25 in Fig. 3). The above described features are of prime importance because the half-width t_0, the response time τ, and, therefore, the ratio t_0/τ can be easily varied in experiment (in contrast to the coupling strength γd).

Figure 4 exhibits the relevant experimental results obtained for our $BaTiO_3$ sample. They correspond to the coupling strength $\gamma d \approx 4.5$. One can clearly see the disappearance of the non-shifted peak, the growth of the time-delayed pulse, and its shift to the right with increasing half-width t_0.

The solid lines in Fig. 5 show the delay time Δt as a function of the input half-width t_0 for three different values of the coupling strength γd. These lines are plotted on the basis of Eq. (5). For $t_0 << \tau$ the delay time is almost a constant that increases rapidly with increasing γd. In the range $t_0 \geq \tau$ the function $\Delta t(t_0)$ experiences a clear growth. The filled squares in Fig. 4 are the experimental results for our $BaTiO_3$ sample. They show a fairly good agreement with the theoretical predictions for $\gamma d = 4$, especially in the range of small half-width t_0. For $t_0 \geq 20$ s the experimental values of Δt are noticeably larger than those predicted by the theory. Possibly, this is due to the effects of light absorption and light-induced scattering.

Lastly, we consider shortly the case of the local photorefractive response, $\gamma = i\gamma''$. No steady-state spatial amplification occurs here during two-wave coupling and amplification of the light pulses is possible only due to the transient effects. An example of photorefractive medium with the local response is $LiNbO_3$:Fe crystals with dominating photovoltaic charge transport [13].

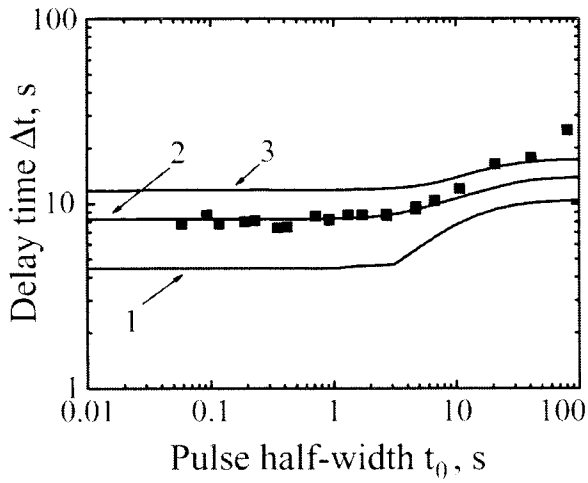

FIG. 5: Dependence of the delay time Δt on the input pulse half-width t_0 for the nonlocal response; the curves 1, 2, and 3 are plotted for $\gamma d = 3$, 4, and 5, respectively. The filled squares are experimental data for $\tau \cong 3$ s.

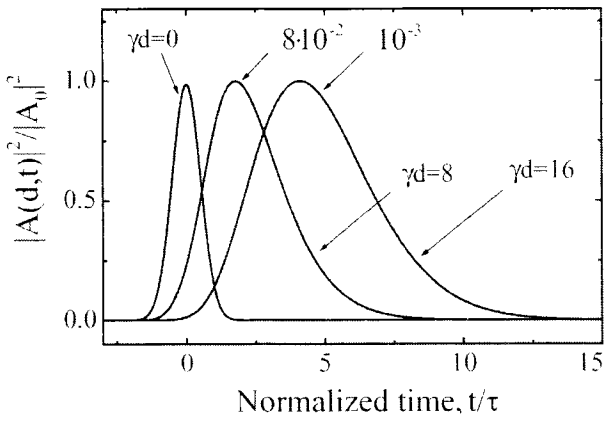

FIG. 6: The output pulse shape for the local response, $t_0/\tau = 1$ the lines 1, 2, and 3 are plotted for $\gamma d = 0$, 8, and 16, respectively. The numbers $8\cdot10^{-2}$ and 10^{-3} are the normalization factors.

Figure 6 shows representative time dependences of the normalized output intensity $|A(d,t)|^2 / |A_0|^2$. The presence of the time delay, which is correlated with the spatial amplification, is clearly seen from these data. Note that the output intensity depends here only on the absolute value of the rate coefficient γ but not on the sign of its imaginary part. Considerably larger values of the coupling strength γd (compared to the case of nonlocal response) are needed to reach equally strong amplification and delay effects. Experiments with PR crystals showing the local response are currently in progress.

Summary

We have studied theoretically and experimentally spatial amplification of light pulses in photorefractive media with nonlocal and local nonlinear response. It is shown that the output pulse is characterized by a considerable time delay with respect to the input signal. This delay grows rapidly with increasing coupling strength and raises slowly with increasing input pulse width. The effective velocity of pulse propagation in the nonlinear medium is extremely small as compared to the light speed. Furthermore, this velocity is controlled by the input pump intensity. The transition to the time delayed regime occurs critically in the range of small input pulse width. The found nonlinear effects can be useful for information processing and material characterization.

Acknowledgments

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