Collective effects in the frictional interface

Oleg Braun

Institute of Physics NASU, Kiev, Ukraine

http://www.iop.kiev.ua/~obraun

in collaboration with

Michel Peyrard (ENS de Lyon)

Erio Tosatti (SISSA & ICTP Trieste)

Dmitry Stryzheus (IoP NASU)

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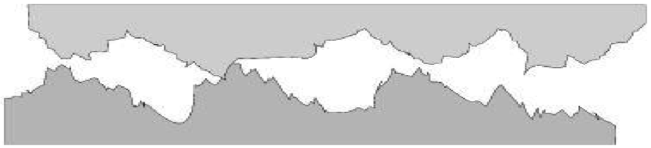
Outline

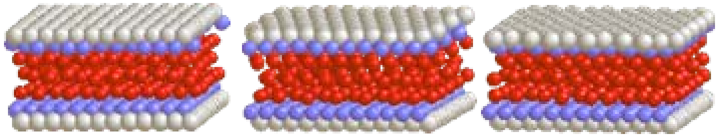
I. EQ (earthquakelike) model & ME (master equation) approach
II. Elastic instability (stick-slip versus smooth sliding)
III. Interaction between contacts (elastic correlation length)
IV. MF (mean field) ME in near zone
V. Crack in the frictional interface as a solitary wave
VI. Conclusion

Nonhomogeneous frictional interface

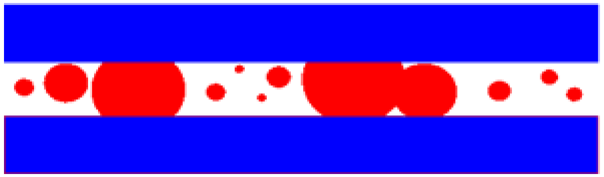
- dry friction:
  contact of rough surfaces



- dry or lubricated friction:
  contact of polycrystalline substrates

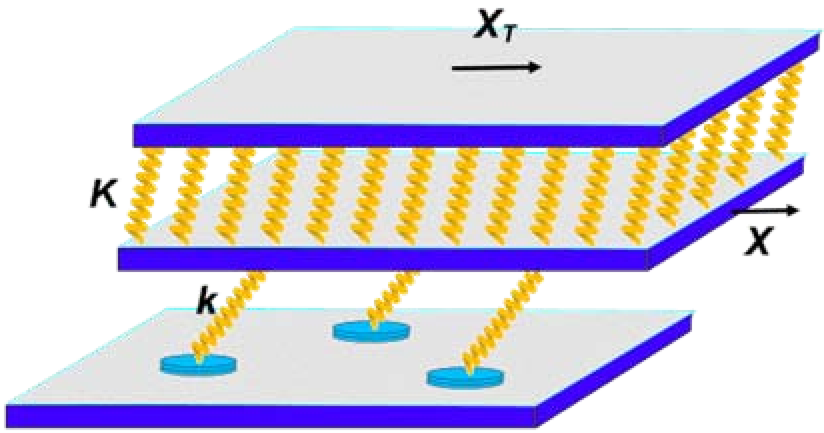


- lubricated friction:
  Lifshitz-Slözov coalescence



I. EQ model & ME approach

The earthquakelike (EQ) model



$P_c(x_s)$ – probability distribution of the thresholds $x_{si}=f_{si}/k_i$ at which the contacts break

$Q(x;X)$ – distribution of the stretchings x_i when the top substrate is at a position X

As the top stage moves, the surface stress at any junction increases, $f_i(t)=k_i x_i(t)$, where $x_i(t)$ is the shift of the i-th junction from its unstressed position.
A single junction is pinned whilst $f_i(t)<f_{si}$, where f_{si} is the static friction threshold for it.
When the force reaches f_{si}, a rapid local slip takes place, during which the local stress drops.
Then the junction is pinned again, and the whole process repeats itself.

Numerics: cellular automaton algorithm

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The master equation (ME) approach

$Q(x;X)$ - the distribution of the stretchings x_i when the bottom of the slider is at X.
$P_c(x_s)$ - probability distribution of values of the thresholds x_{si} at which contacts break.
$R(x)$ - probability distribution of values of the displacements x for "newborn" contacts.

Consider a small displacement $\Delta X > 0$ of the bottom of the solid block.
It induces a variation of the stretching x_i of the asperities which has the same value ΔX.
The displacement X leads to three kinds of changes in the distribution $Q(x;X)$:

$$Q(x; X + \Delta X) = Q(x - \Delta X; X) - \Delta Q_-(x; X) + \Delta Q_+(x; X)$$

(1) the first term is just the shift due to the global increase of the stretching;
(2) some contacts break because the stretching exceeds the maximum that they can stand:

$$\Delta Q_-(x; X) = P(x)\,\Delta X\, Q(x; X), \quad P(x) = P_c(x) \Big/ \int_x^\infty d\xi\, P_c(\xi)$$

the number of contacts to be broken $= N_c P_c(x) \Delta X$

the number of still unbroken contacts ($/N_c$)

(3) those broken contacts form again after a slip:

$$\Delta Q_+(x; X) = R(x) \int_{-\infty}^\infty d\xi\, \Delta Q_-(\xi; X)$$

Finally, with $\Delta X \to 0$ we get the integro-differential equation:

$$\frac{\partial Q(x;X)}{\partial x} + \frac{\partial Q(x;X)}{\partial X} + P(x)\, Q(x; X) = R(x) \int_{-\infty}^\infty d\xi\, P(\xi)\, Q(\xi; X)$$

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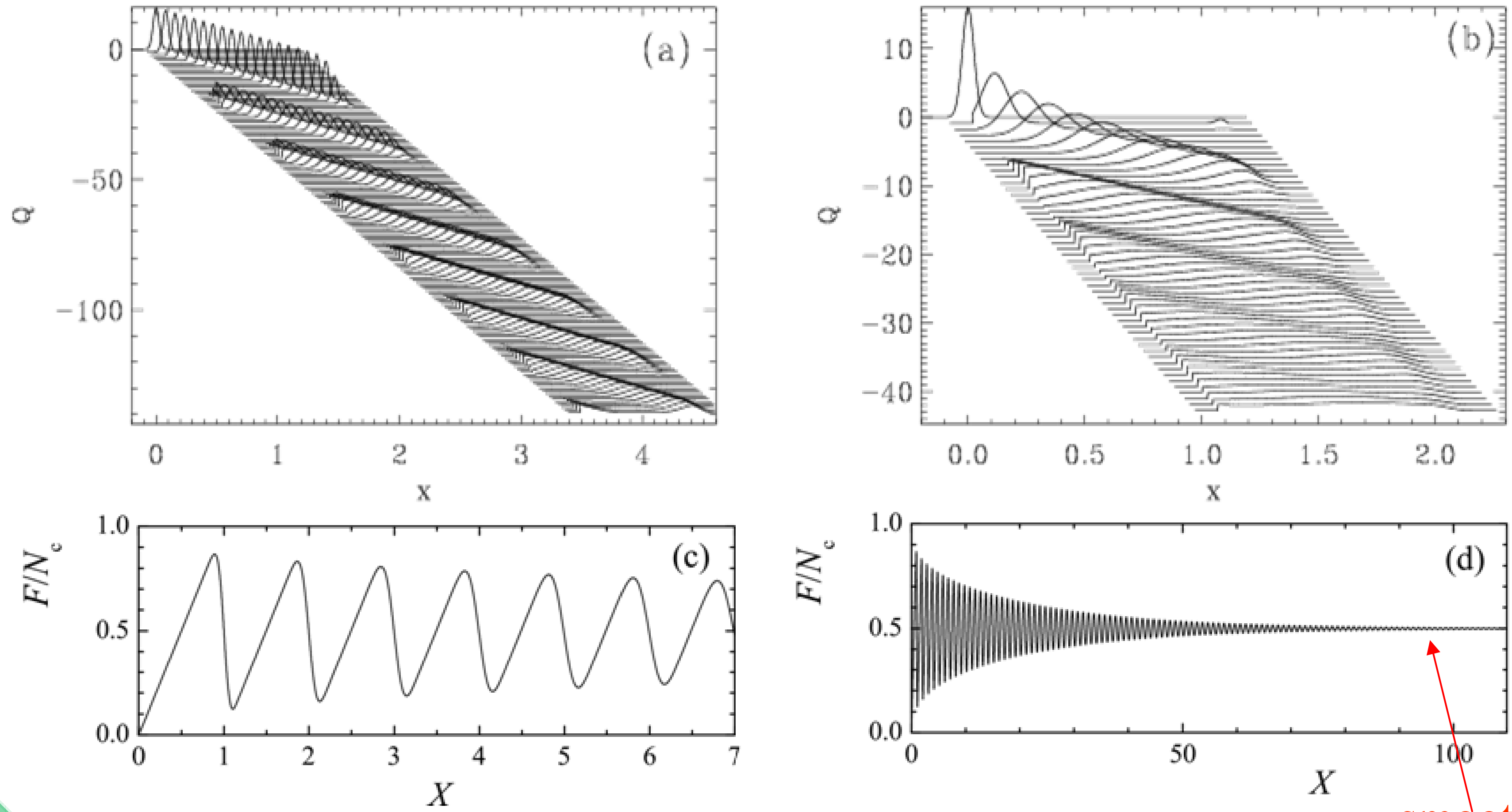
I. EQ model & ME approach

Solution:

short times — long times

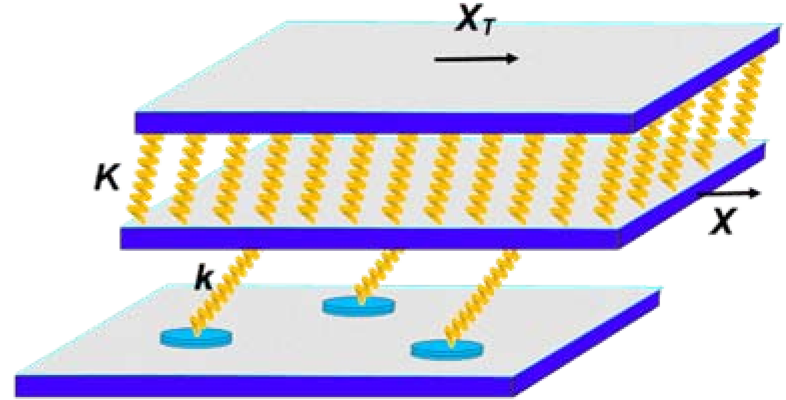


smooth sliding !

$$F(X) = N_c \langle k \rangle \int dx \; x \, Q(x;X)$$

$$P_c(x) = \text{Gauss } (\bar{x}_s = 1, \, \sigma_s = 0.05), \; Q_{\text{ini}}(x) = \text{Gauss } (\bar{x}_{\text{ini}} = 0, \, \sigma_{\text{ini}} = 0.025)$$

II. Elastic instability



The force at the substrate/lubricant interface $F = K(X_d - X)$ (*) must be equal to the force $F(X)$ from friction contacts. When X_d and X increase, the substrate remains stationary as long as $dX_d/dX > 0$.

$dX_d/dX = 0$, or $F'(X) \equiv dF(X)/dX = -K$ (**) just defines the maximal displacement X_m which the contacts can sustain; a larger displacement will break all the contacts simultaneously, and at this moment all contacts will reform.

$$K^* = -\max F'(X) \approx Nk\,(f_s - f_b)\,/\Delta f_s$$

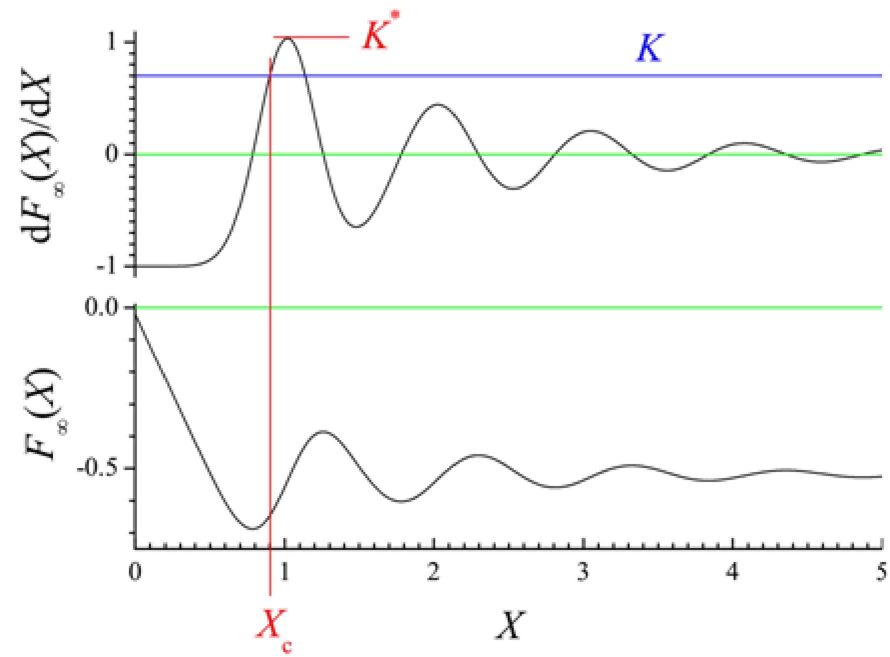
OR:

The total potential energy of the sliding interface plus the elastic substrate is

$V(X) = \int_0^X dX'\, F(X') + \frac{1}{2} K(X - X_d)^2$;

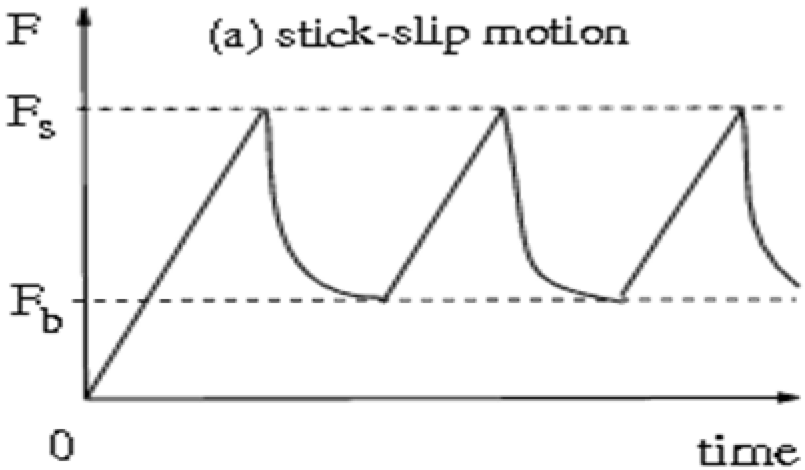
then Eq.(*) $\leftrightarrow V'(X) = 0$;

it is stable if $V''(X) > 0$, so that the unstable displacement is defined by $V''(X) = 0 \leftrightarrow$ Eq.(**)

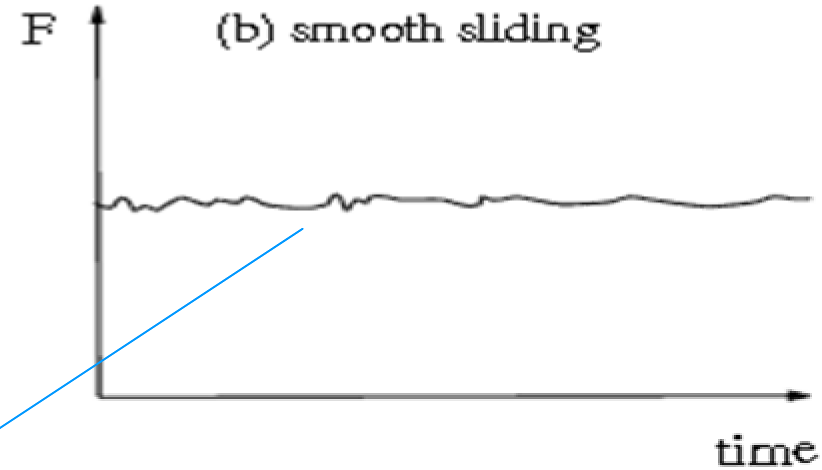


II. Elastic instability

$K < K^*$: stick-slip



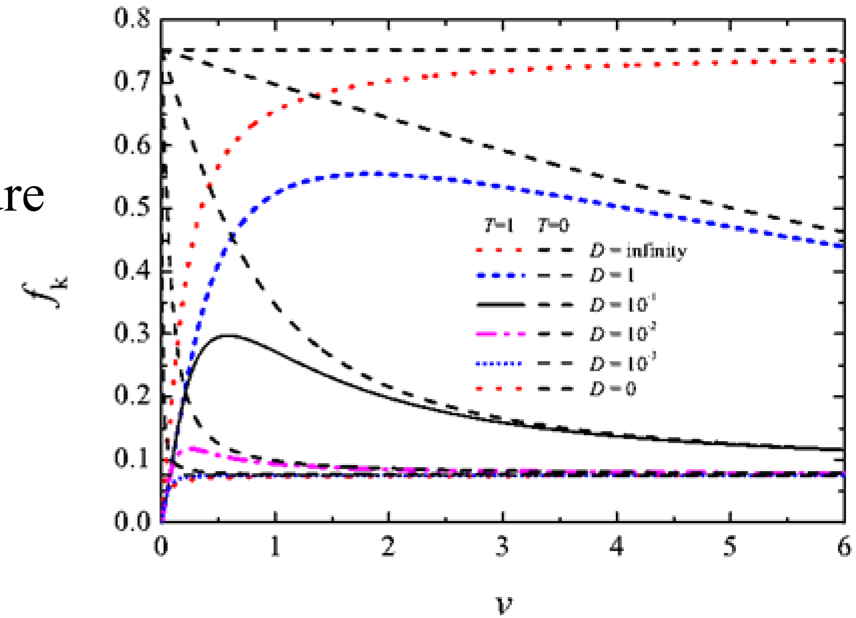
$K > K^*$: smooth sliding



smooth sliding:

$f_k(v)$ increases at small v due to temperature

$f_k(v)$ decreases at large v due to aging



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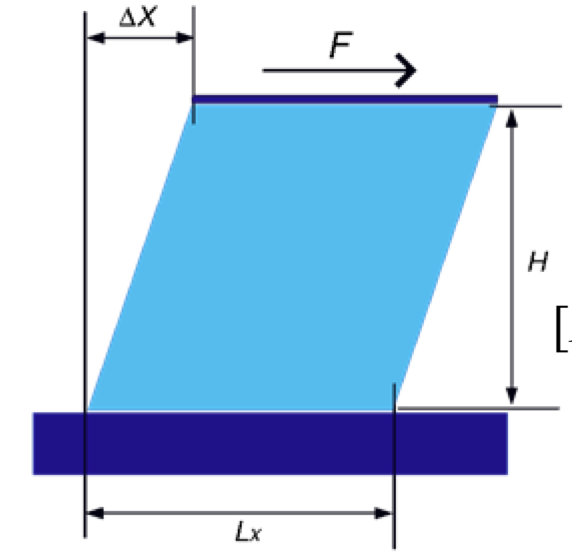
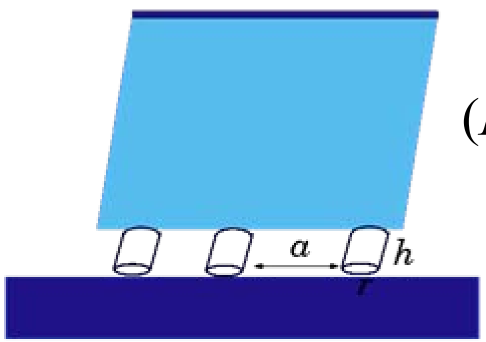
II. Elastic instability

Estimation



$$K = F/\Delta X = [E/2(1+\sigma)](L_x L_y/H)$$



$$K_s = Nk = (L_x L_y/a^2)\,(E r_c)\,(r_c/h)^3$$

$$K/K_s = 0.16\,(a^2/H r_c)\,(h/r_c)^3$$

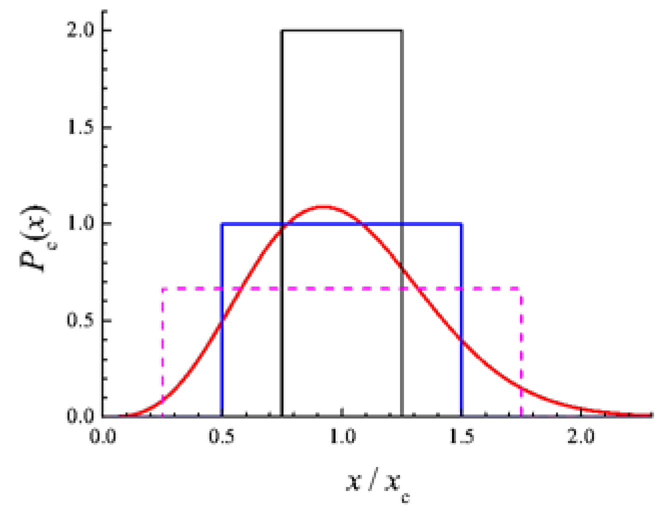
$a \sim 10^2$ μm

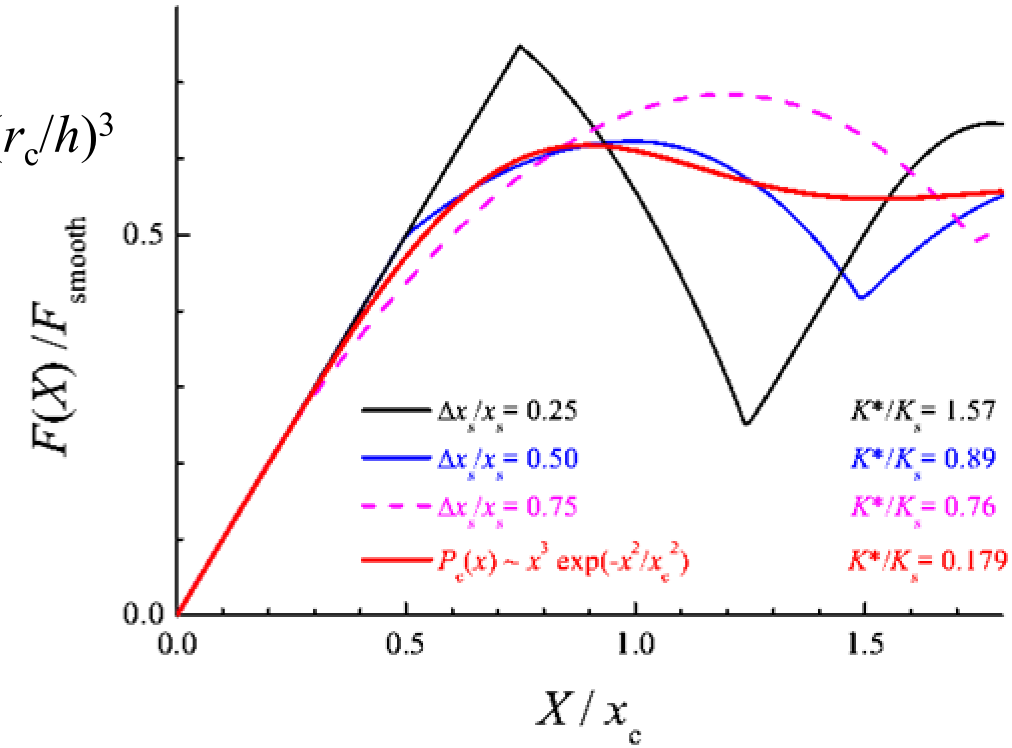
$r_c \sim 1$ μm

$H \sim 1$ cm

$h \sim r_c$

$$K/K_s \sim 0.1$$





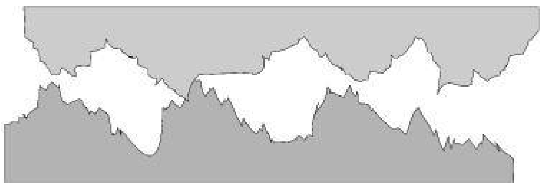
II. Elastic instability ?

$P_c(x)$

dry friction: contact of rough surfaces

dry or lubricated friction: contact of polycrystalline substrates

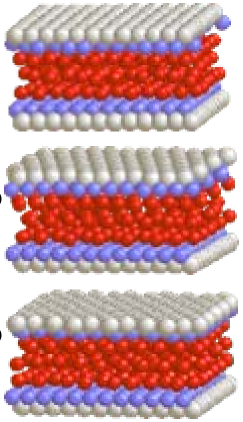
lubricated friction: Lifshitz-Slözov coalescence

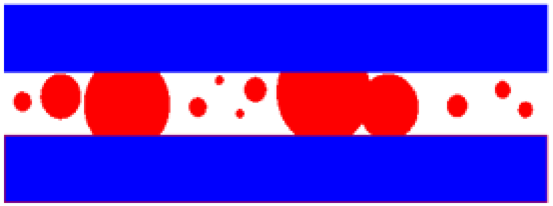


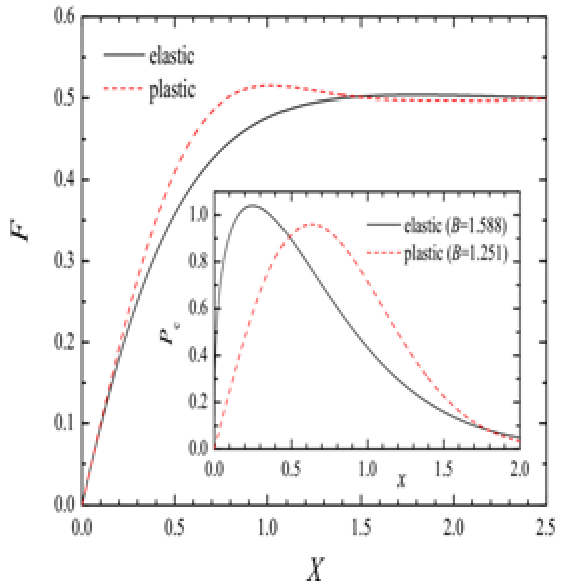
$\varphi = 0$

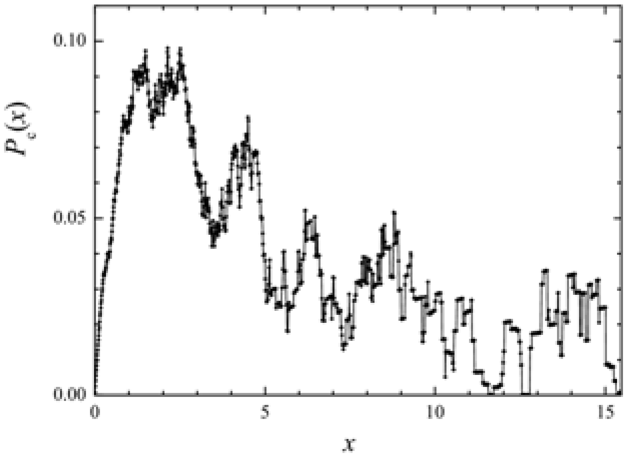
$\varphi = 15°$

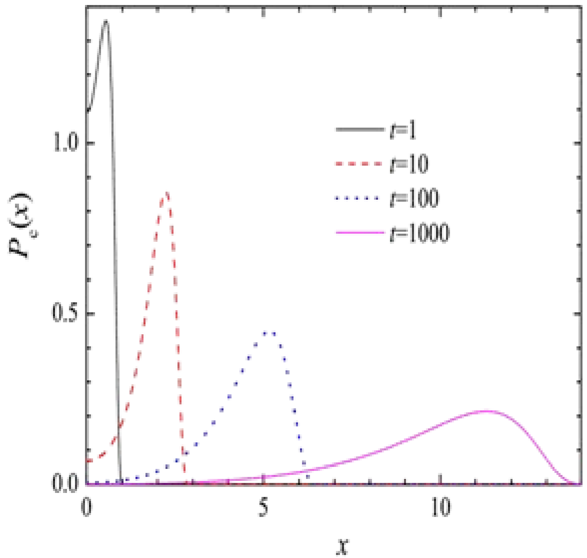
$\varphi = 30°$











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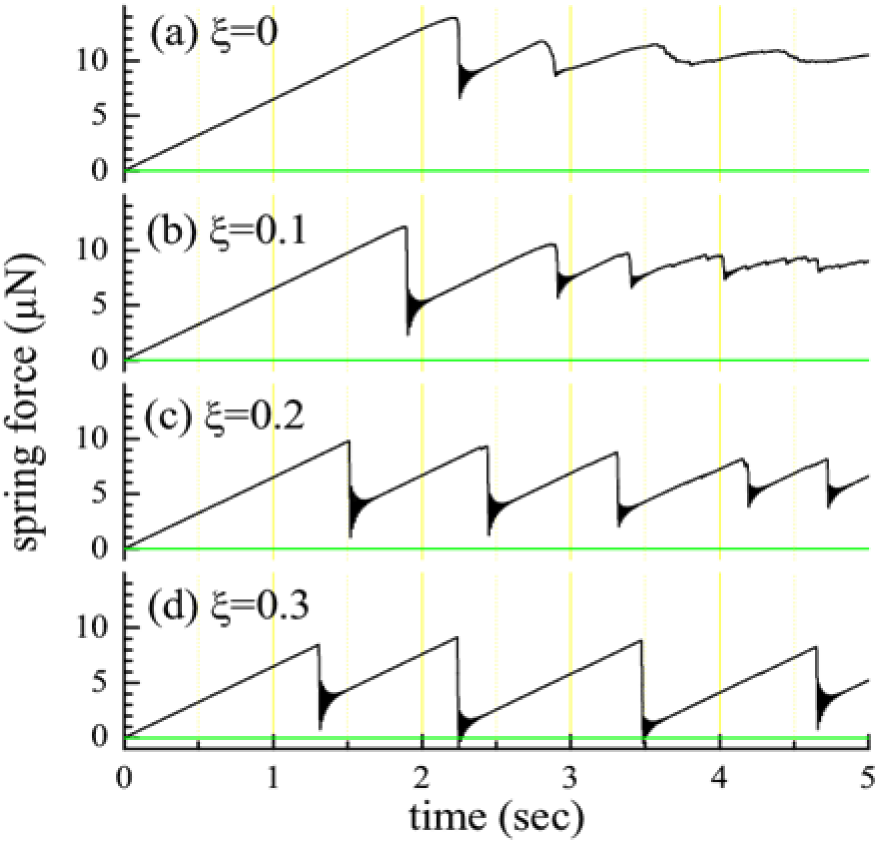
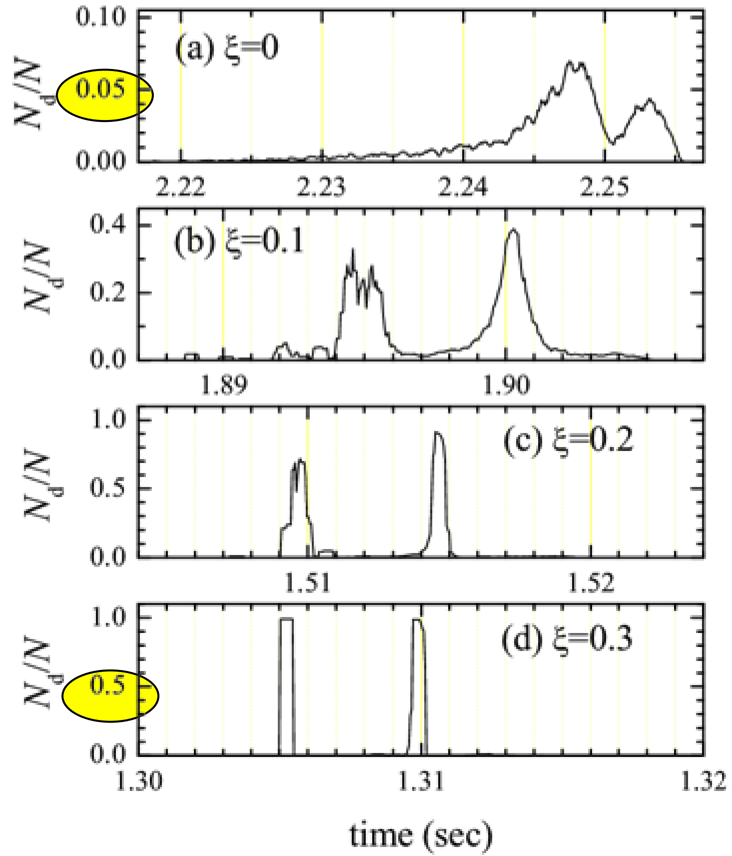
III. Interaction between contacts

EQ simulation

The interaction between the contacts works roughly in the same way as the dispersion Δf_s:
the stronger is the interaction, the wider is the range of model parameters where stick-slip occurs.
System kinetics with increasing interaction $\xi \sim f_{\rm int}/f_s$:
the system quickly goes to smooth sliding for noninteracting contacts (a),
but demonstrates stick-slip for a strong interaction (d).





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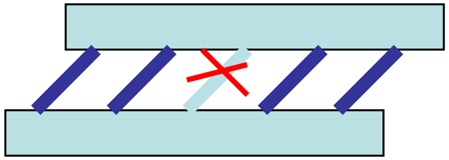
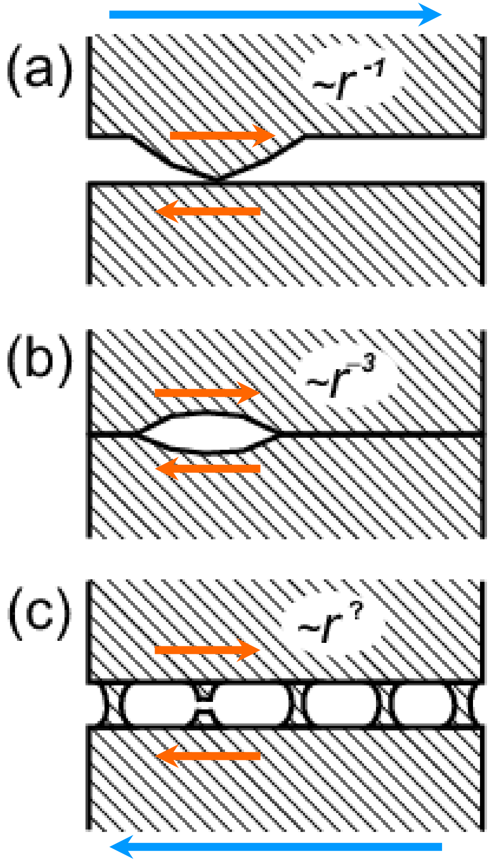
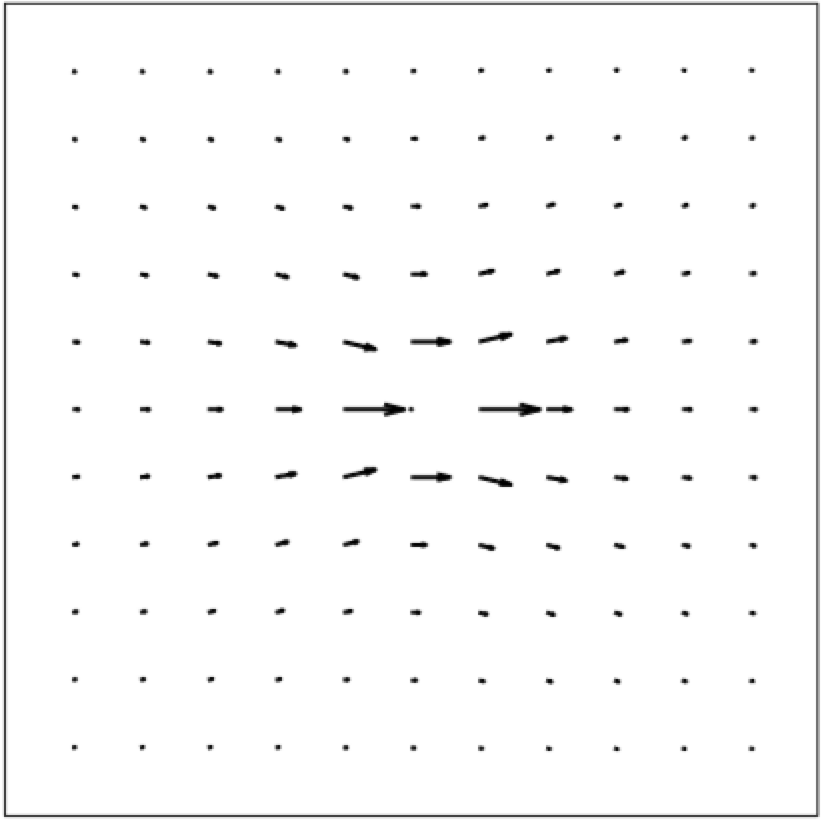
III. Interaction between contacts

The law of interaction





$\gamma_1 = k/(Ea) = 0.06$



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III. Interaction between contacts **Formulas**

Contacts act on the top substrate by the forces $\mathbf{f}_i \equiv \{f_{ix}, f_{iy}, f_{iz}\}$. They produce displacements $\mathbf{u}_i^{(\mathrm{top})}$ of the (bottom) surface of the top substrate. The vectors $\mathbf{U}^{(\mathrm{top})} \equiv \{\mathbf{u}_i^{(\mathrm{top})}\}$ and $\mathbf{F}_t \equiv \{\mathbf{f}_i\}$ are coupled by $\mathbf{U}^{(\mathrm{top})} = \mathbf{G}^{(\mathrm{top})}\mathbf{F}_t$. Elastic Green tensor $\mathbf{G}^{(\mathrm{top})}$ for a semi-infinite substrate (Landau and Lifshitz):

$$\begin{aligned} G_{ix,jx} &= g(r_{ij})[2(1-\sigma) + 2\sigma x_{ij}^2/r_{ij}^2] \\ G_{ix,jy} &= 2g(r_{ij})\,\sigma x_{ij} y_{ij}/r_{ij}^2 \\ G_{ix,jz} &= -g(r_{ij})(1-2\sigma)\,x_{ij}/r_{ij} \\ G_{iz,jx} &= -G_{ix,jz} \\ G_{iz,jz} &= 2g(r_{ij})(1-\sigma)\,, \end{aligned} \tag{1}$$

where $x_{ij} = x_i - x_j$, $g(r) = (1+\sigma)/(2\pi E r)$, and σ and E are the Poisson ratio and Young modulus of the top substrate, respectively.

In equilibrium, the forces that act from the contacts on the bottom substrate, must be equal to $-\mathbf{F}_t$ according to third Newton law. These forces lead to displacements of the (top) surface of the bottom substrate, $\mathbf{U}^{(\mathrm{bottom})} = -\mathbf{G}^{(\mathrm{bottom})}\mathbf{F}_t$. Thus, the relative displacements at the interface due to elastic interaction between the contacts are determined by the relation
$\mathbf{U} \equiv \mathbf{U}^{(\mathrm{top})} - \mathbf{U}^{(\mathrm{bottom})} = -\mathbf{G}\mathbf{F}$, where $\mathbf{F} = -\mathbf{F}_t$ and $\mathbf{G} = \mathbf{G}^{(\mathrm{top})} + \mathbf{G}^{(\mathrm{bottom})}$.

III. Interaction between contacts

Formulas

The forces and displacements are coupled by the diagonal matrix (the contacts' elastic matrix) $\mathbf{K}$, $K_{i\alpha,\,j\beta} = k_{i\alpha}\delta_{ij}\delta_{\alpha\beta}$ $(\alpha, \beta = x, y, z)$: $\mathbf{F} = \mathbf{K}\,(\mathbf{U}_0 + \mathbf{U})$, where $\mathbf{U}_0$ defines a given stressed state. The total force at the interface, $\mathbf{f} = \sum_i \mathbf{f}_i$, must be compensated by external forces applied to the substrates, e.g., by the force $\mathbf{f}^{(\text{ext})} = \mathbf{f}$ applied to the top surface of the top substrate if the bottom surface of the bottom substrate is fixed.

Combining, we obtain $\mathbf{F} = \mathbf{K}\,(\mathbf{U}_0 - \mathbf{G}\mathbf{F})$, or

$$\mathbf{F} = \mathbf{B}\mathbf{K}\mathbf{U}_0\,, \quad \text{where} \quad \mathbf{B} = (\mathbf{1} + \mathbf{K}\mathbf{G})^{-1}.$$

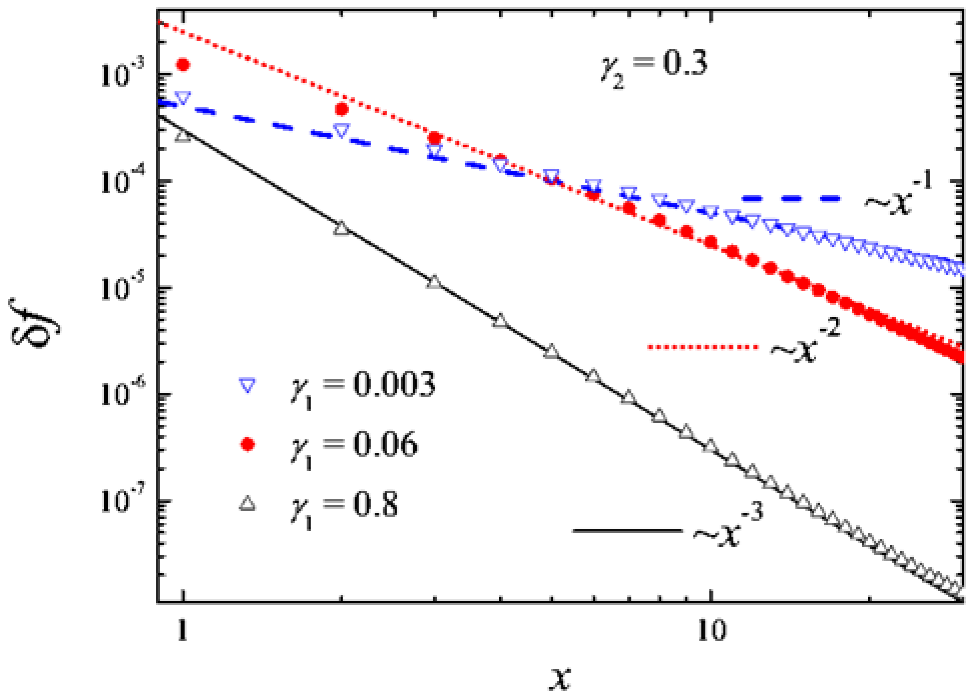
If one changes the contact elastic matrix, $\mathbf{K} \to \mathbf{K} + \delta\mathbf{K}$, then the interface forces should change as well, $\mathbf{F} \to \mathbf{F} + \delta\mathbf{F}$. Then, $\delta\mathbf{F} = (\delta\mathbf{B})\mathbf{K}\mathbf{U}_0 + \mathbf{B}(\delta\mathbf{K})\mathbf{U}_0$, $\delta\mathbf{B}$ may be found from the equation $\delta[\mathbf{B}\,(\mathbf{1}+\mathbf{K}\mathbf{G})] = (\delta\mathbf{B})(\mathbf{1}+\mathbf{K}\mathbf{G})+\mathbf{B}(\delta\mathbf{K})\mathbf{G} = 0$. Therefore, finally we obtain:

$$\delta\mathbf{F} = \mathbf{B}\,\delta\mathbf{K}\,(\mathbf{1} - \mathbf{G}\mathbf{B}\mathbf{K})\mathbf{U}_0\,.$$

Now, if we remove the i^*th contact by putting $\delta k_{i\alpha} = -k_{i\alpha}\delta_{ii^*}$ and then calculate the resulting change of forces on other contacts, we can find a response of the interface to the break of a single contact as a function of the distance $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_{i^*}$ from the breaking contact.

III. Interaction between contacts

Results:



$$\gamma_1 = k\,/\,(Ea)$$

$$\gamma_2 = r_c\,/\,a$$

Elastic correlation length: $\lambda_c = a\,(Ea\,/\,k)$

$r < \lambda_c$: $\delta f(r) \sim r^{-1}$
rigid slider
MF

$r > \lambda_c$: $\delta f(r) \sim r^{-3}$
deformable slider
solitonic wave

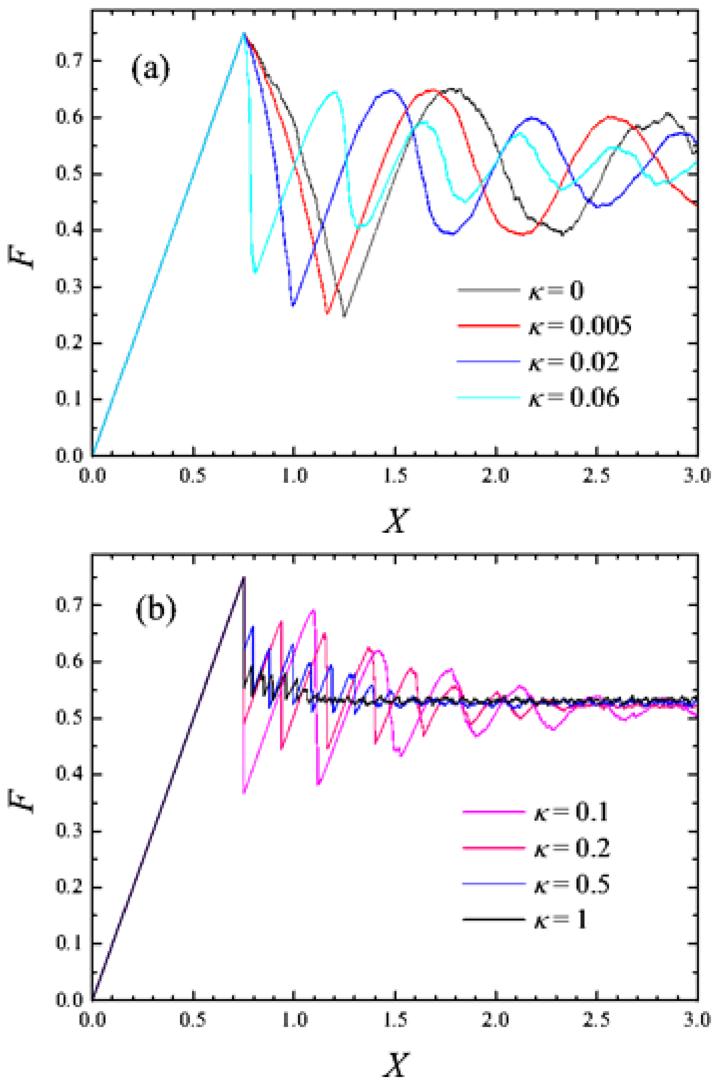
C. Caroli & Ph. Nozieres, *Eur. Phys. J. B* **4** (1998) 233
O.M. Braun & D. Stryzheus, *unpublished*

IV. Mean Field (MF) ME

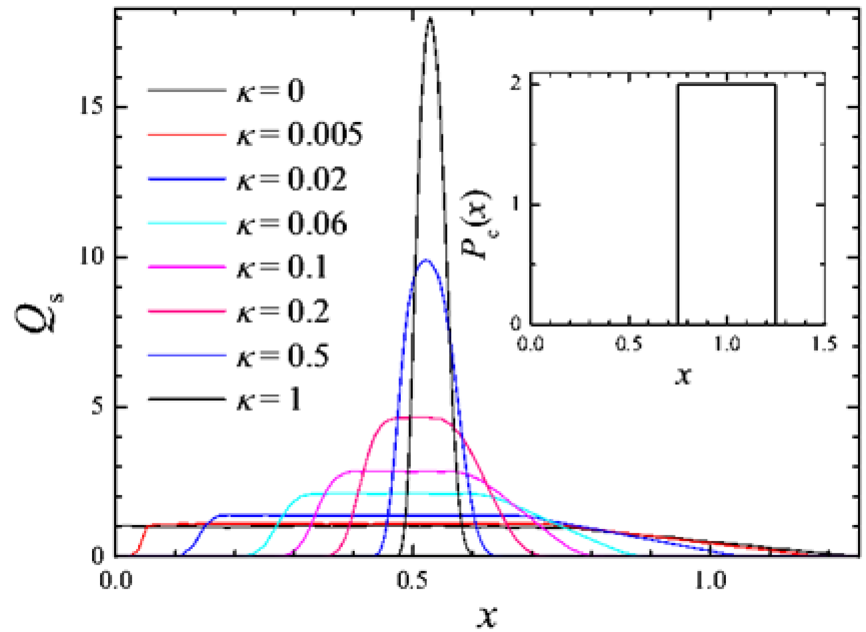
Near zone: EQ simulation

$$\Delta f_{ij} = \boldsymbol{\kappa}\, k x_c \,(u_j - u_i) \,/\, |\boldsymbol{r}_j - \boldsymbol{r}_i|$$

Force versus displacement



The steady state distribution of stretchings

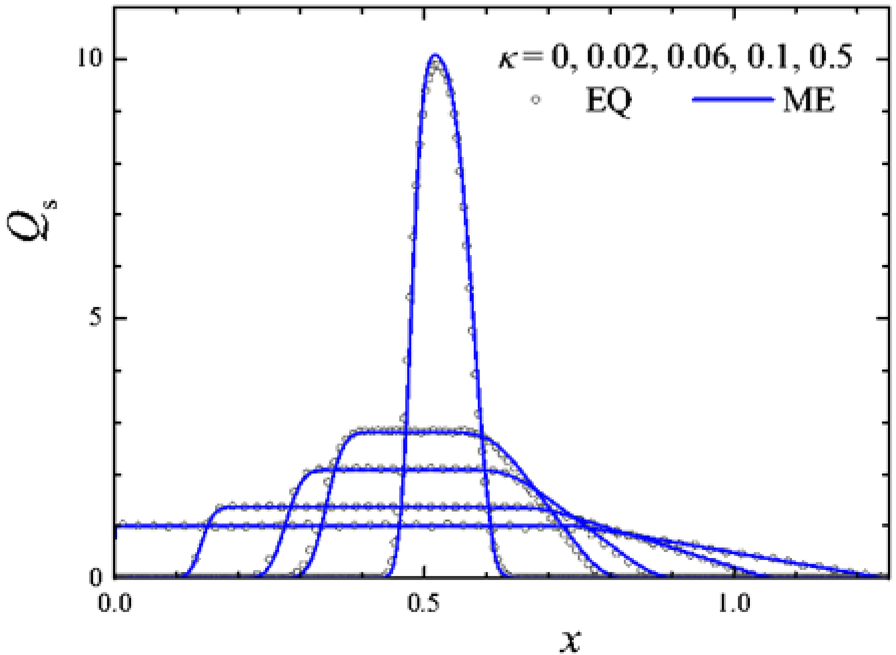


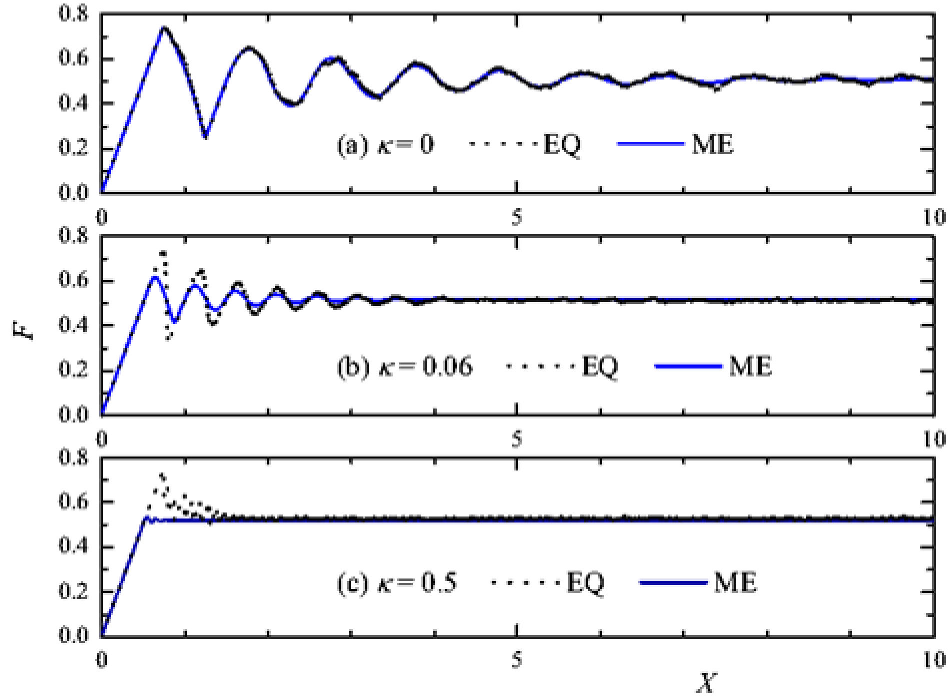
1. In the steady state, the interaction results in **shrinking of the final distribution** $Q_s(x)$. At high values of κ, the distribution approaches to a narrow Gaussian one.
2. At the onset of sliding, the rate of $F(X)$ decreasing grows with κ; therefore, **the elastic instability becomes stronger** due to contact-contact interaction.
3. For large enough strength of interaction, $\kappa > \kappa_c \sim 0.1$, many contacts break simultaneously at the onset of sliding, and the force $F(X)$ drops abruptly.

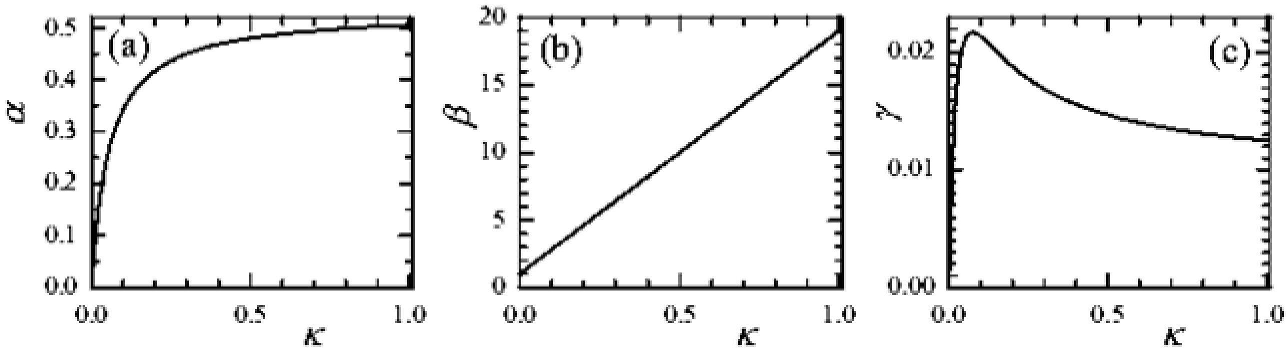
IV. Mean Field ME

Near zone: smooth sliding

$$P_c(x) = \beta P_{c0}[\beta(x - \alpha x_c)], \;\; R(x) = \text{Gauss}\,(x - \alpha x_c, \gamma x_c)$$



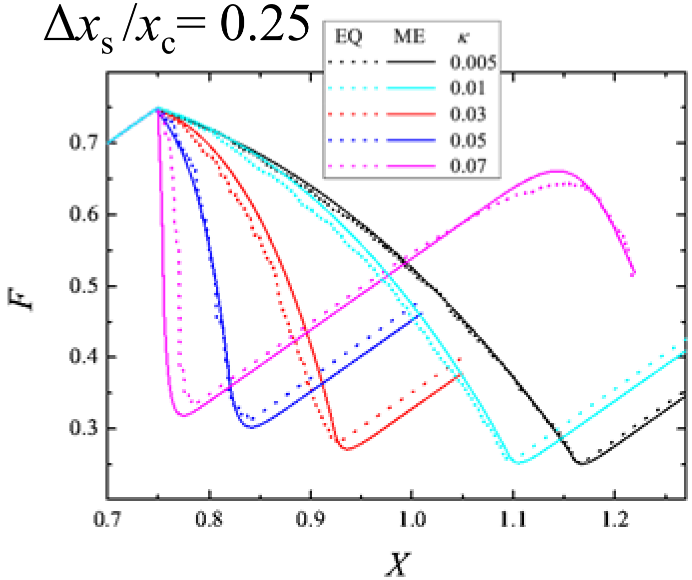




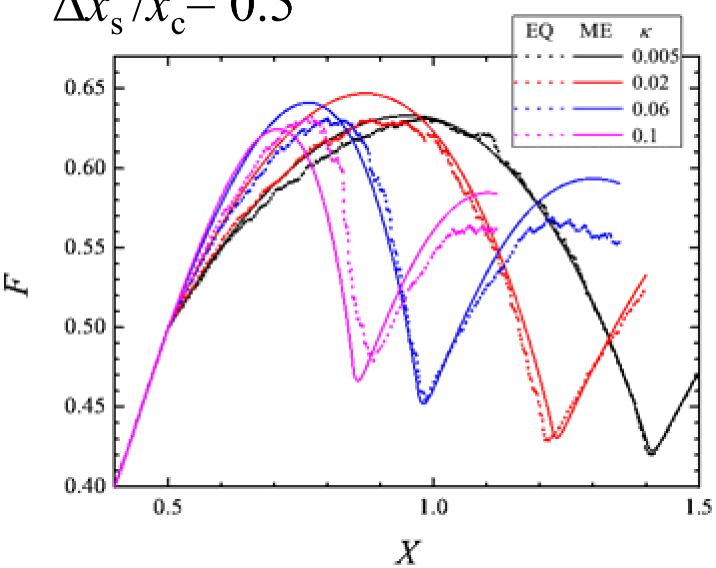
IV. Mean Field ME

Near zone: onset of sliding

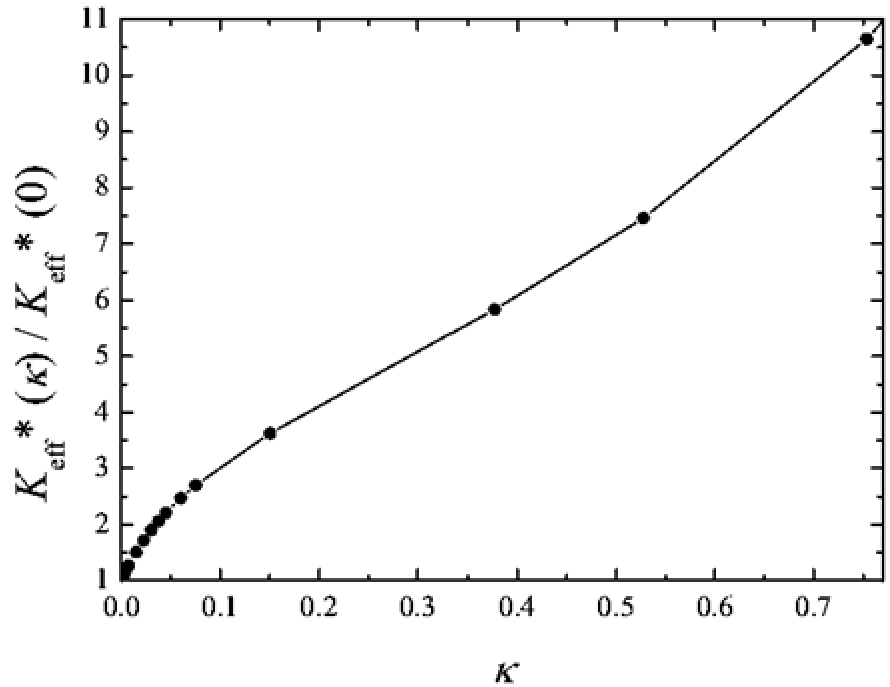
$\Delta x_s / x_c = 0.25$



$\Delta x_s / x_c = 0.5$



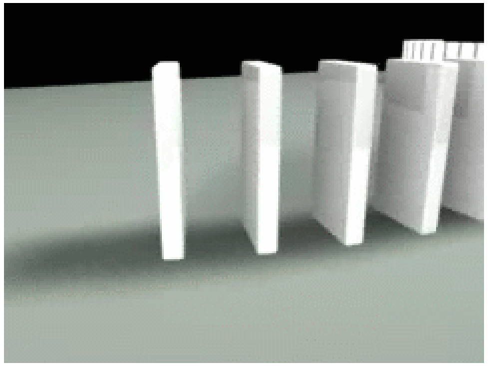
$$P_{ci}(x) = \mathrm{N}\, x^{\varepsilon} P_{c0}[\beta_0(x - \alpha_0 x_c)]$$



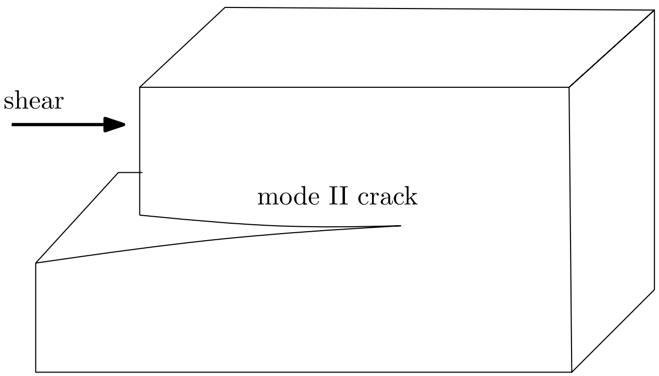
The effective interface stiffness K^*_{eff} (normalized on the noninteracting value $K^*/K_s = 0.179$) as a function of the strength of interaction κ for a realistic threshold distribution $P_{c0}(x) \propto x^3 \exp(-x^2/x_c^2)$

V. Crack as a solitary wave

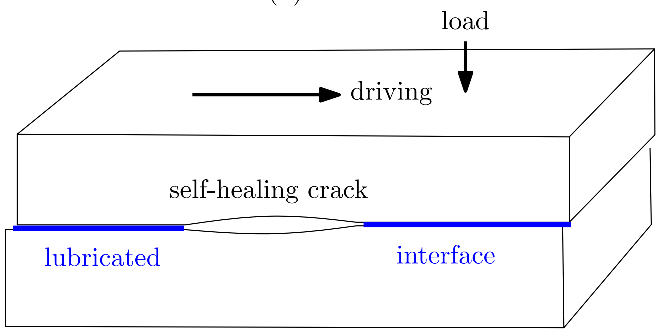
Idea: domino effect



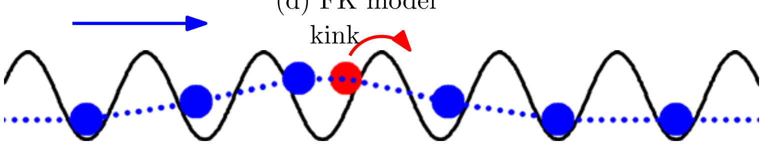
(a) Fracture



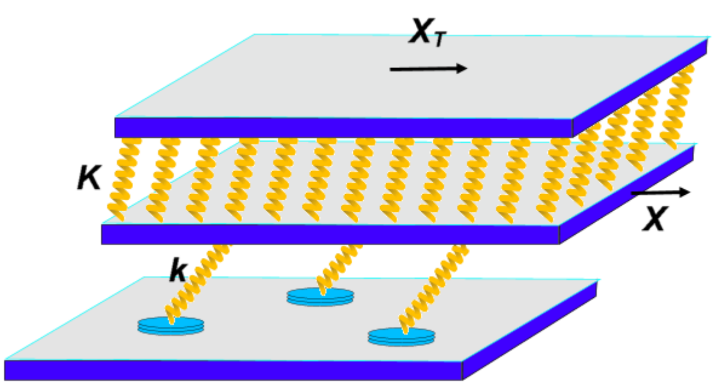
(b) Friction



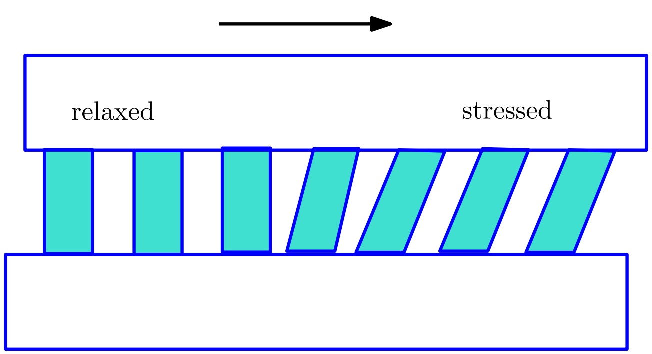
(d) FK model



(c) EQ model

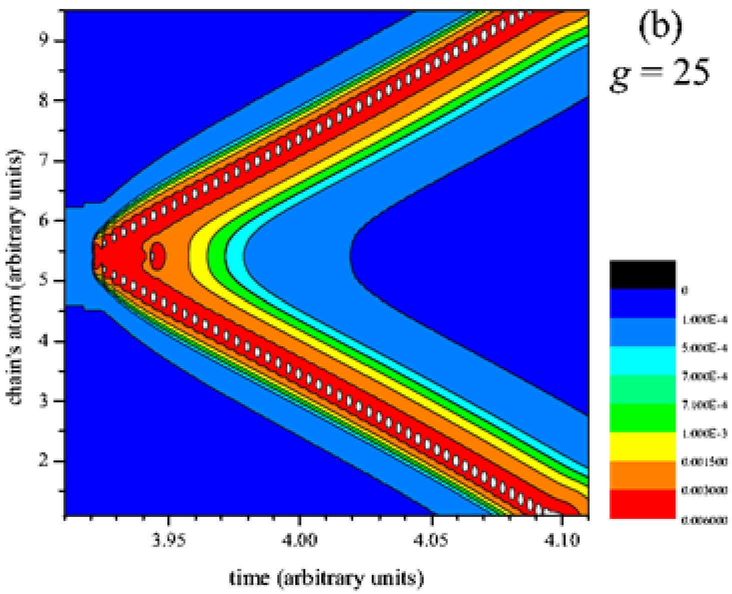


(e) The model

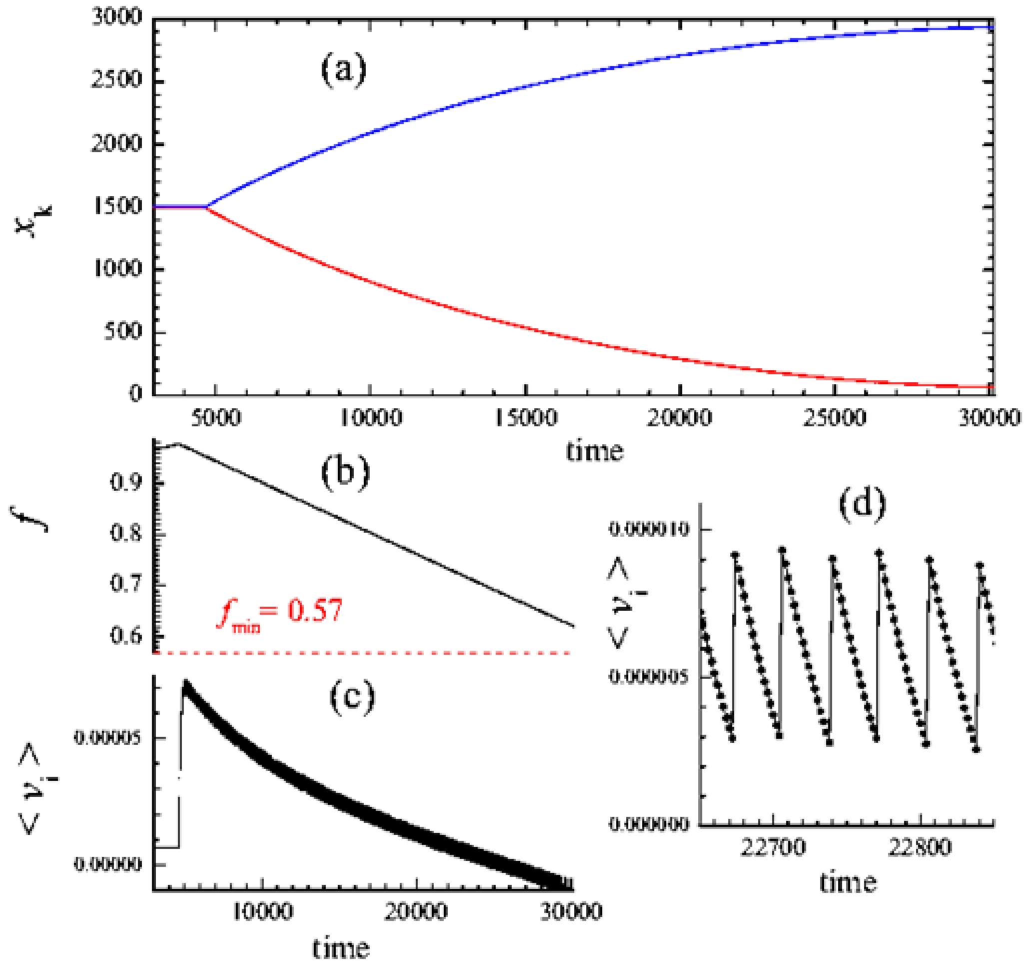


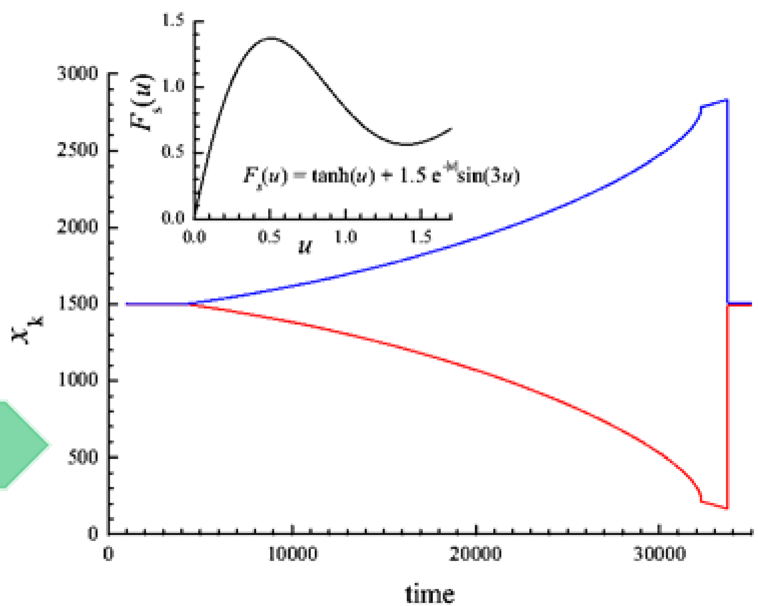
V. Crack as a solitary wave

FK: simulation



1D model: sawtooth interaction





V. Crack as a solitary wave

FK: formulas

$$m\ddot{u}_i + m\eta\dot{u}_i - g(u_{i+1} + u_{i-1} - 2u_i) + F_s(u_i) + Ku_i = f(t) = Kv_d t$$

Define the function $\mathcal{F}(u) = F_s(u) + Ku - f$

Boundary conditions: right part is unrelaxed, $u_R = f/(k+K)$

left is relaxed, $u_L = (f + ku_c)/(k+K)$

Continuum approximation ($i \to x = ia$):

$$m\eta u_t - a^2 g u_{xx} + \mathcal{F}(u) = 0, \quad \mathcal{F}(u)|_{x\to\pm\infty} = 0$$

Look for a solution in the form of a wave of stationary profile

(the solitary wave) $u(x,t) = u(x - vt)$

Solution: $f_{\min} = \left(\frac{1}{2}k + K\right) u_c, \; f_{\max} = (k+K)\, u_c$

Kink velocity as a function of the driving force: at low velocities

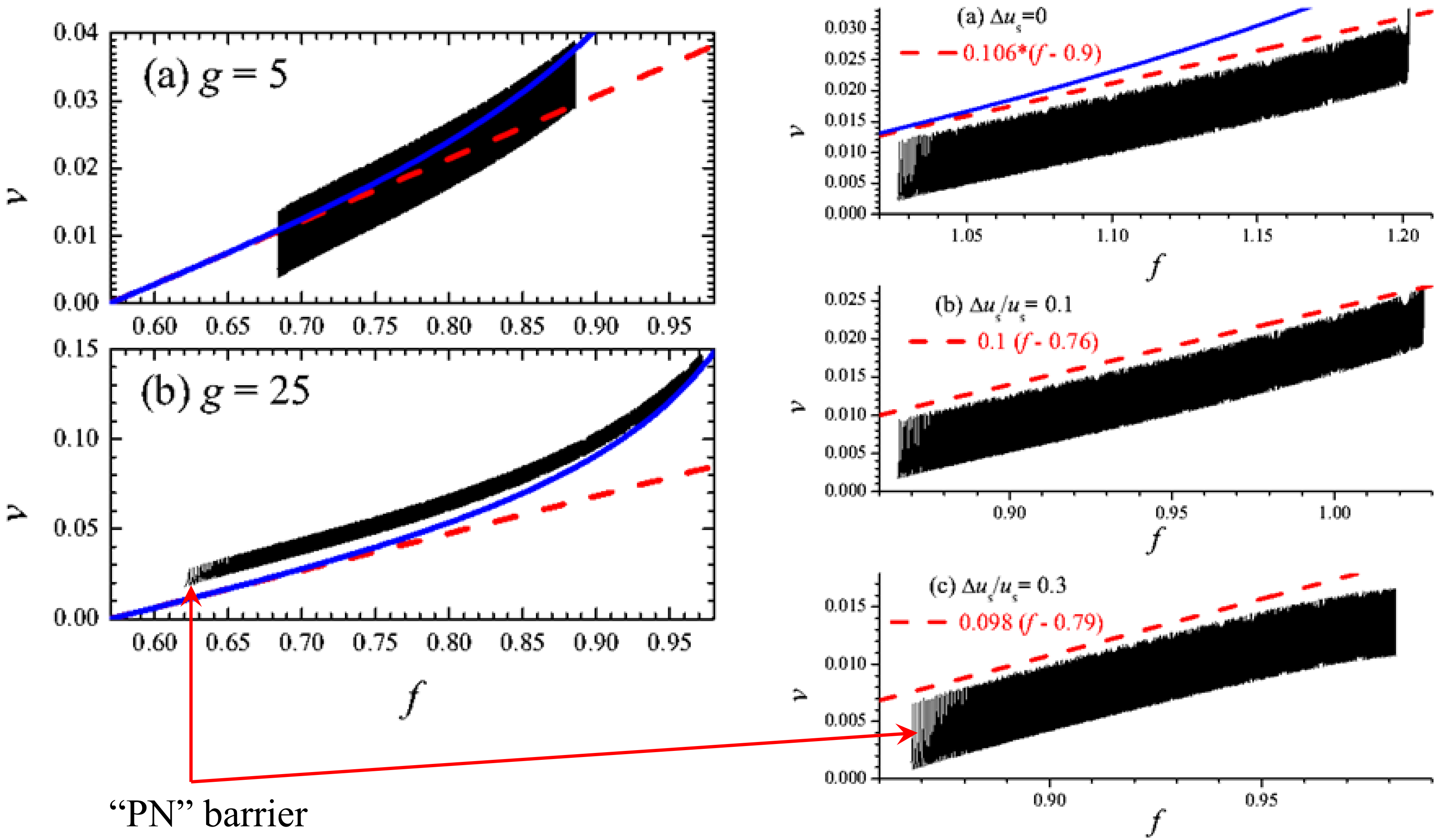
$$v \approx (f - f_{\min})/m_k\eta\,, \quad m_k = m \bigg/ \frac{4a}{u_c}\sqrt{\frac{g}{k}\left(1 + \frac{K}{k}\right)} \tag{1}$$

at high velocities

$$v \approx c_0 \bigg/ \sqrt{1 + \frac{m\eta^2(f_{\max} - f)}{k(k+K)u_c}} \tag{2}$$

V. Crack as a solitary wave FK: *v*(*f*)

simulation versus analytics



V. Crack as a solitary wave   FK generalizations

Using analogy with the FK model:

1. A role of **disorder** and **defects**: (a) defects may **stimulate** kinks creation;
   (b) kink's propagation may be slowed down up to its complete **arrest** due to pinning on the defects.

2. The driven FK model exhibits **hysteresis** when the force increases/decreases.

3. $T>0$: the sliding kinks will experience an additional damping,
   while the immobile kinks will slowly move (**creep**) due to thermally activated jumps.

4. The fast driven kink begins to **oscillate** due to excitation of its shape mode,
   and then, with the further increase of driving, the kink is destroyed.

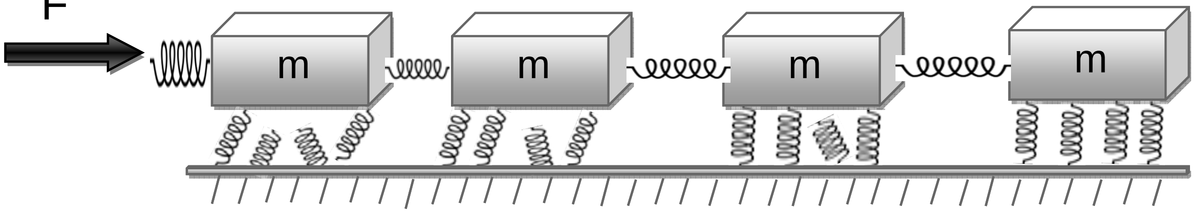
5. If the interaction between the atoms is nonlinear and stiff enough,
   the FK model admits the existence of **supersonic** kink.

6. A large number of works is devoted to different generalizations of the FK model to **2D system**.
   For example, if kinks attract one another in the transverse direction, they unite into a line (dislocation) which moves as a whole (or due to secondary kinks).

7. **Nonuniform shear stress**: for given boundary conditions (which depend on the experimental setup) one has to calculate the stress field → the driving force $f(x,y)$ in the FK-EQ model.
   Thus, finally we come to a **self-consistent** problem:
   the whole system is described by elastic-theory equations with complex boundary conditions –
   at the frictional interface they are determined through solution of the FK-EQ model
   (where the driving term comes from the elastic equations in turn).

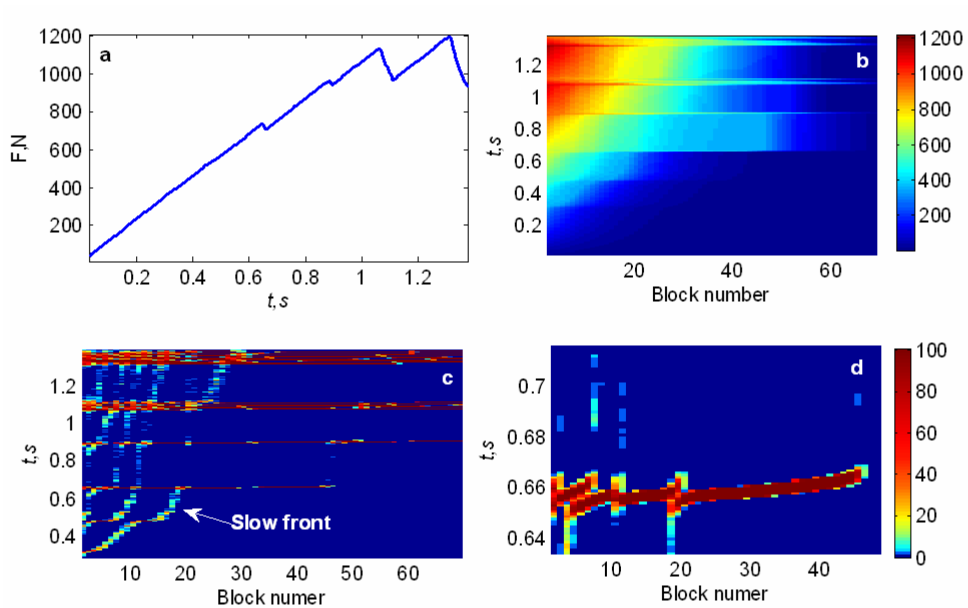
V. Crack as a solitary wave

Simulation: Onset of sliding



(a) loading curve
$F(t)$

(c) distribution of fraction of attached contacts as a function of the block number j and time t.
The regions with attached contacts = blue color, detached = red color.



Bars set up a correspondence between the colors and the force in Newton (b) and the fraction of detached contacts in % (c, d).

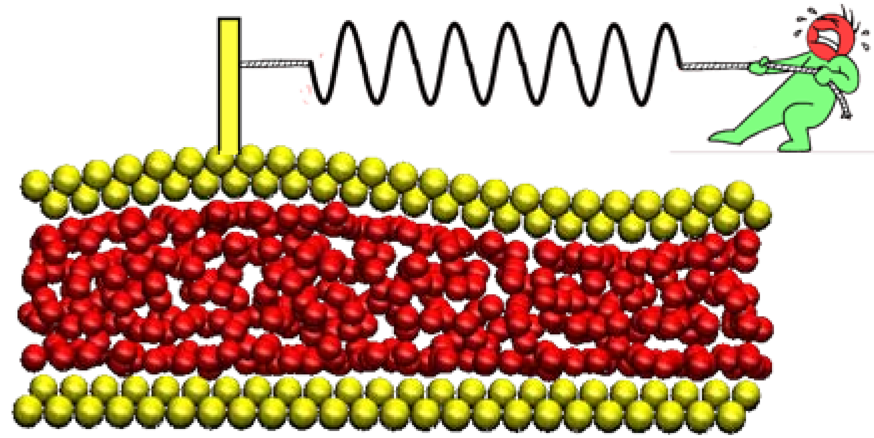
(b) distribution of elastic forces in the slider as a function of the block number j and time t.
The unstressed and stressed regions are displayed by blue and red colors.

(d) enlarged view of the fast detachment front from (c) showing an excitation of a secondary Rayleigh front by the slow fronts

experiment: S.M. Rubinstein, G. Cohen & J. Fineberg, *Nature* **430** (2004) 1005; *prl* **98**, 226103 (2007)
simulation: O.M. Braun, I. Barel & M. Urbakh, *Phys. Rev. Lett.* **103** (2009) 194301

VI. Conclusion

1. Interaction: elastic correlation length $\lambda_c = a^2 E / k$
2. Near zone – shrinking, enhancing of elastic instability
3. Far zone – collective modes (solitonic waves)



ME: O.M.Braun & M.Peyrard, *Phys. Rev. Lett.* **100** (2008) 125501 *"Modeling friction on a mesoscale: Master equation for the earthquakelike model"*; *Phys. Rev. E* **82** (2010) 036117 *"Master equation approach to friction at the mesoscale"*; *Phys. Rev. E* **83** (2011) 046129 *"Dependence of kinetic friction on velocity: Master equation approach"*
EQ: O.M. Braun & J. Roder, *Phys. Rev. Lett.* **88** (2002) 096102 *"Transition from stick-slip to smooth sliding: An earthquakelike model"*; O.M. Braun, I. Barel, and M. Urbakh, *Phys. Rev. Lett.* **103** (2009) 194301 *"Dynamics of transition from static to kinetic friction"*; O.M. Braun & E. Tosatti, *Europhys. Lett.* **88** (2009) 48003 "Kinetics of stick-slip friction in boundary lubrication"; O.M. Braun & E. Tosatti, *Philosophical Magazine* **91** (2011) 3253 *"Kinetics and dynamics of frictional stick-slip in mesoscopic boundary lubrication"*; O.M. Braun & N. Manini, *Phys. Rev. E* **83** (2011) 021601 *"Dependence of boundary lubrication on the misfit angle between the sliding surfaces"*; N. Manini & O.M. Braun, *Phys. Lett. A* **375** (2011) 2946 *"Crystalline misfit-angle implications for solid sliding"*; O.M. Braun & D.V. Stryzheus, *unpublished* *"Characteristic lengths at the sliding interface"*
reviews: O.M. Braun & A.G. Naumovets, *Surf. Sci. Reports* **60** (2006) 79 *"Nanotribology: Microscopic mechanisms of friction"*; O.M. Braun, *Tribology Letters* **39** (2010) 283 *"Bridging the gap between the atomic-scale and macroscopic modeling of friction"*