Transition from Stick-Slip to Smooth Sliding: An Earthquakelike Model

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We present a detailed study of an earthquakelike model that exhibits a "transition" from stick-slip motion to smooth sliding at a velocity of the order of those observed in experiments. This contrasts with the many previous microscopic models in which the transition velocity is many orders of magnitude too large. The results show that experimentally observed smooth sliding at the macroscopic scale must correspond to microscopic-scale stick-slip motion.

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The problem of understanding tribological behavior is very important [1,2]. Experiments show that in all cases when the static frictional force is nonzero, $f_s > 0$, the system exhibits a transition from a stick-slip motion at low velocities and spring constant to smooth sliding at high velocities and/or a stiff system. This situation is typical for most tribological systems except the ones with a liquid lubricant. Therefore, it is observed in all systems at low enough temperatures, when the lubricant is solid, or when the lubricant is squeezed out from the contact region. In experiments, the critical velocity of the transition from stick-slip to smooth sliding is typically of the order $v_c \sim 1\ \mu$m/s [3]. In most cases, smooth sliding is a more desired regime (one exception is bowing a violin); thus the problem emerges as how to avoid or reduce the stick-slip regime. However, to do this, first of all one has to understand the mechanisms of stick-slip motion and the transition to smooth sliding.

A phenomenological model of this problem was developed in a series of papers [4]. It is based on a "contact-age function" which depends on the previous history of the system. The model leads to excellent agreement with experiments, if the model parameters are suitably chosen. Unfortunately, it remains purely phenomenological, in that the corresponding equations cannot be derived from a microscopic-scale simulation.

There are a number of molecular dynamics (MD) studies, and other microscopic models of the problem (e.g., see [2] and references therein). The lubricant film between the two solids may be very thin due to normal pressure between the sliding objects [5]. The thin film solidifies [2,6], and the system is pinned for low driving forces, f. If the force increases, the pinned state will remain until f reaches a critical stress f_s. At this point one of the solid blocks starts to move, with the local drift velocity increasing abruptly from zero to $v_s = v(f_s)$. At the onset of sliding the lubricant film may or may not fluidize, depending on the nature of the lubricant. If the amplitude of interatomic interaction within the lubricant is larger than the interaction between the lubricant and the solid substrate, the solid-sliding regime is observed; otherwise the lubricant melts at the onset of sliding [7]. Then, if f is reduced below f_s, the block does not return directly to the pinned state but continues to slide until f reaches some lower critical stress f_b, where it returns to the pinned state. Hysteresis may appear due to two different mechanisms: inertial effects [7] (similar to the bistability of an underdamped driven atom in an inclined periodic potential), or the melting/freezing of the lubricant [8,9]. In both cases, however, the velocity on decreasing of the force, $v_b = v(f_b)$, is of *atomic-scale order,* $v_b \sim 1$–10 m/s. The reason is that the thin confined film is in close contact with the solid substrates; thus the rate of energy exchange between the lubricant and the substrates is of atomic-scale value. One can show [10] that taking into account the large mass of the sliding block does not change the results (contrary to what was supposed in [8]), because the pinning begins with the stopping of the first layer of the block, and the kinetic energy of the sliding block is converted into a wave propagating away from the interface which is finally absorbed in the substrate (see also [11]). If the top block is driven with a velocity v through an attached spring, we obtain smooth sliding for $v > v_c$ and stick-slip sliding for $v < v_c$ as shown in Fig. 1. Always, however, $v_c \sim v_b$ is on the atomic scale, e.g., $v_c \sim 10^{-2}c$ (c is the sound speed), which is 6 orders of magnitude higher than the experimentally observed values. Thus, the microscopic mechanism of the stick-slip to smooth sliding transition observed in MD simulation has little in common with the experimentally observed macroscopic one.

In the present Letter we propose a variant of an earthquakelike model which demonstrates stick-slip behavior at low velocities and changes to "smooth" sliding at high v. The "transition" takes place at $v_c \lesssim a/\tau$, where a is the average distance between junctions and τ is an "aging" time of a single junction. Reasonable values for these parameters (e.g., $a \sim 10^{-6}$–10^{-3} m and $\tau \sim 1$–10^3 s [1,2]) lead to experimentally observed values of v_c. The model predicts that experimentally observed smooth sliding actually corresponds to atomic-scale stick-slip motion of individual junctions and that the transition itself is a smooth one, if one increases the resolution of the velocity increments.

Model.—We use a 2D variant of the Burridge-Knopoff (BK) spring-block model of earthquakes [12], similar to

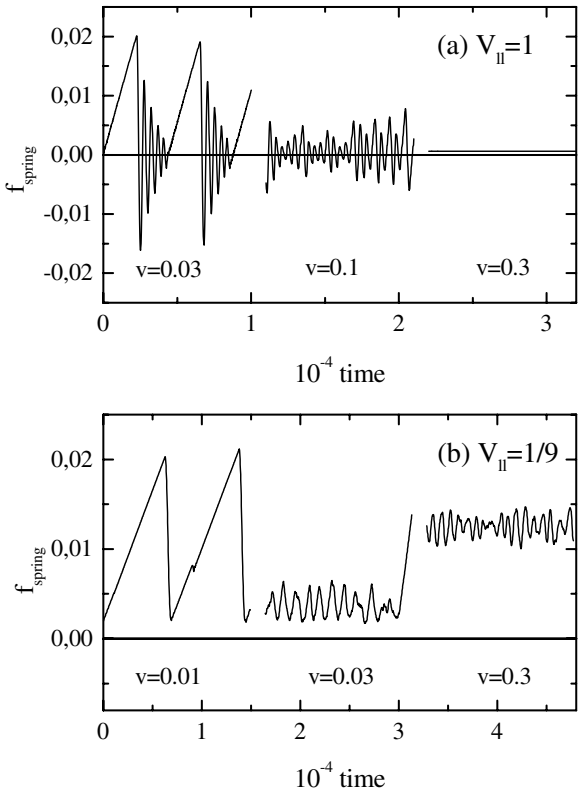

FIG. 1. Spring force vs time when a spring with the elastic constant $k = 3 \times 10^{-4}$ is attached and moves with a constant velocity v. (a) The case of a "solid" lubricant ($V_{ss} = 3$, $V_{sl} = 1/3$, $V_{ll} = 1$) for velocities $v = 0.03$, 0.1, and 0.3; (b) a "liquid" lubricant ($V_{ss} = 3$, $V_{sl} = 1/3$, $V_{ll} = 1/9$) for velocities $v = 0.01$, 0.03, and 0.3. The dependencies were obtained by MD simulation of the system described in [7]. V_{ss}, V_{sl}, and V_{ll} stand for the strength of the Lennard-Jones interaction inside the substrate, between the substrate and lubricant atoms, and inside the lubricant, respectively. The units are dimensionless.

that studied by Olami, Feder, and Christensen (OFC) [13]. Let the two blocks touch one another at (point) junctions which pin the relative position of the blocks. The junctions form an array $\{r_i\}$ randomly distributed in 2D space, $r_i = r_{i0} + (\xi_i - 0.5)\Delta r$, where $i = 1, \ldots, N$ numerates the junctions, r_{i0} corresponds to a uniform distribution (the triangular lattice), ξ_i is a standard random number, and the parameter Δr describes the amplitude of randomness. The junctions interact elastically via springs of strength k_{ij}; all junctions are connected through springs of strength k with the fixed bottom block and coupled frictionally with the top block moving with a constant velocity v. The elastic constants are $k \sim \langle k_{ij} \rangle \sim \rho c^2 a$, where ρ is the mass density of the block, c is the transverse sound velocity, and $a = \langle r_{ij} \rangle$ [14]. The potential energy $V(r)$ of the elastic interaction between two defects separated by a distance r in a solid [15] as well as on a crystal surface [16] follows the law $V(r) \approx k_{\rm int}/r^3$, where $k_{\rm int}$ is a parameter describing the elastic properties of the block. Thus, k_{ij} are determined by the expressions $k_{ij} = 3k_{\rm int}/|r_j - r_i|^5[5(x_j - x_i)^2/(r_j - r_i)^2 - 1]$.

We study a scalar variant of the model, when only the x component of the force is considered. Let $u_i(t)$ represent the shift of the ith junction from its nonstressed position. The local force $f_i(t)$ associated with each junction is the sum of the force from the bottom block, $f_i^{(b)}(t) = ku_i(t)$, where $u_i(t) = u_i(t_0) + v(t - t_0)$ due to frictional coupling with the top block, and the elastic forces from other junctions, $f_i^{(\rm int)}(t) = -\sum_j k_{ij}[u_j(t) - u_i(t)]$. As the top stage moves, the surface stress at any junction increases continuously. A single junction is pinned while $f_i(t) < f_s(t)$. When the force on a given junction, i, reaches the critical value $f_i(t) = f_s(t)$, this junction starts to slide. At this point, a rapid local slip takes place, during which the local stress in the block drops to the value f_b. The sliding takes a time $t < 10^{-10}$ s [2,7] and thus can be considered as an instantaneous one. The coordinate u_i of the relaxing junction (instantly) changes to the new position $u_i = (f_b + \sum_{j \neq i} k_{ij}u_j)/(k + \sum_{j \neq i} k_{ij})$. The slip of one junction redefines the forces on its neighbors; this can result in further slips (an avalanche); the triggered "earthquake" will stop when there are no junctions left with a force above the threshold. Then the junctions are pinned again, and the whole process repeats itself. As the initial configuration we use random shifts of all junctions, $u_i(0) = \xi_i \Delta x_{\rm ini}$.

Following the discovery of self-organized critical (SOC) behavior in a BK-type model [17], many studies of this type were performed [18]. If we set $\Delta r = 0$, our model reduces to the OFC model [13] except that we use the triangular lattice instead of the square one. For periodic boundary conditions (PBC) the steady state of the OFC model is always periodic [19]. However, for open boundary conditions (OBC) the model exhibits SOC behavior; the probability distribution $P(s)$ of the number of relaxations s in a single avalanche follows the power law $P(s) \propto s^{-\chi}$ with the exponent χ continuously varying with $k_{\rm int}$ (or v). However, in both cases the OFC model does not demonstrate a transition from stick-slip motion to smooth sliding.

The main new feature of our model is that we use the "age-function" idea of the phenomenological models [4], i.e., we assume that the static frictional force depends continuously on the time of stationary contact of a given junction. We used a simple exponential dependence (let $t = 0$ correspond to the moment when the junction is pinned),

$$f_{si}(t) = f_s + (f_{\rm sm} - f_s)[1 - \exp(-t/\tau)]. \quad (1)$$

Notice that $f_{si}(t)$ reinitializes every time the junction relaxes and, thus, it is different for different junctions.

Without loss of generality in what follows, we put $k = 1$, $a = 1$, $\tau = 1$, and $f_s = 1$. In our simulations we typically used $f_{\rm sm} = 2f_s$ which corresponds, e.g., to the squeezing of a two-layer lubricant film into a one-layer configuration [7], $f_b = 0.1$, $\Delta r = 0.3$, $\Delta x_{\rm ini} = 1$, $k_{\rm int} = 0.1$ (recall $12k_{\rm int} \sim k$), and $N \gtrsim 10^3$ (for real systems $N/A \sim 10^2$–10^5 cm^{-2}, e.g., see [1]), although we varied all these parameters over a wide range.

A rather comprehensive study of the 1D BK-type model for the parameters suitable for the frictional process and

with incorporation into the model the $f_s(t)$ dependence (1) has been done by Persson [14]. The important result of his study is that this type of model can explain the logarithmic time dependence of relaxation processes at nonzero temperatures: it is due to thermally activated processes which occur near the sharp cutoff at $f = f_s$ in the distribution of surface stresses. Unfortunately, the resulting $f(v)$ dependences do not reproduce the experimental ones very well. As we show below, the reason lies in the one dimensionality of his model.

Simulation results.—We studied several variants of the model, such as 1D and 2D models, models with short- and long-range interaction between the junctions, and also a "stimulated" variant of the model, where we took into account that when a junction relaxes, it emits a wave burst which stimulates other junctions to relax, too. All variants were studied for a wide range of model parameters. Finally we came to the conclusion that in order to reproduce typical experimentally observed $f(t)$ dependencies, the "minimal" model must (a) be 2D, (b) incorporate the $f_s(t)$ dependence (1), and (c) have a random spacial distribution of contacts, $\Delta r \neq 0$. A typical dependence of the total frictional force $f(t)$ for different velocities v is presented in Fig. 2a.

When $v \gtrsim 1$, our model behaves similarly to the OFC model. The difference is that, due to the randomness of the junctions' distribution, $\Delta r \neq 0$, the function $f(t)$ shows a complicated, nonperiodic behavior even for PBC, and the distribution of avalanche sizes is exponential both for PBC and OBC as shown in Fig. 3b (the power-law distribution is observed for $\Delta r = 0$ with OBC only). The average size of the avalanches can be estimated in the following way. At t a given (ith) junction, pinned for an "age" τ_i, relaxes. Then the force on a neighboring (jth) junction abruptly increases by the amount $\Delta f_j = [f_{si}(\tau_i) - f_b]b$, where $b = k_{ij}/(k + \sum_{j' \neq i} k_{ij'})$. For the triangular lattice with the parameters $k_{ij} \sim k$ we have $\langle b \rangle \approx \frac{1}{7}$. (In 1D one has $\langle b \rangle \approx \frac{1}{3}$.) The nearest neighboring junction, j, will relax, too (and, thus, the avalanche will start), if $f_j(t + 0) = f_j(t - 0) + \Delta f_j \geq f_{sj}(\tau_j)$. In the high-velocity regime we can put $f_{si} \approx f_s$, because $\langle \tau_i \rangle \ll \tau$. The distribution of forces $\mathcal{P}(f_i)$ in this case has a simple form [14]: it is constant for forces within the interval $f_b < f_i < f_s$ and zero outside it (in what follows we assume $f_b = 0$ to shorten the notation).

Thus, the probability p that the jth junction will relax, i.e., that $f_j(t - 0) \geq (1 - b)f_s$, is equal to $p = b$. Because there are six nearest neighbors around the "starting" junction i, the probability to have an avalanche of size $s \geq 2$ is $P_s(2) = 6p$. Then, the jth junction may stimulate its own (five) nearest neighbors to relax; thus the probability to have an avalanche of size $s \geq 3$ is $P_s(3) = P_s(2)5p$. Iterating this process, we have $P_s(s) = P_s(s - 1)\nu p$, or $P_s(s) \propto (\nu p)^s$, where $3 \leq \nu \leq 5$ [$\nu = 5$ if the avalanche forms a one-dimensional nonintersecting curve and $\nu \approx 3$ when the avalanche is compact (2D)]. Using $P_s(s) = \int_s^\infty ds' P(s')$, we obtain for the avalanche distribution $P(s) \propto \exp(-s/\bar{s})$, where the average size of the avalanche is $\bar{s} = -1/\ln(\nu p)$. Taking $\nu = 5$ and $p \approx \frac{1}{7}$, we obtain $\bar{s} \approx 3$ for the triangular lattice. This is in agreement with simulation which yields $\bar{s} \approx 4$ for the $v = 3$ case in Fig. 2. The fluctuations of the total frictional force scale as $\langle f(t) - \langle f(t) \rangle \rangle \propto N^{-1/2}$ with the number of junctions N. Note that similar considerations for the one-dimensional system lead to $\nu = 1$ (the avalanche can expand in one direction only) and $p \approx \frac{1}{3}$, so that $\bar{s} \approx 1$. Hence, for large v we have $\nu p < 1$, so that $\bar{s} < \infty$, so the avalanche cannot occupy the whole system.

When $v \lesssim 1$, the model exhibits stick-slip behavior as can be seen from Fig. 2a [20]. The distribution of avalanche sizes possesses two peaks as shown in Fig. 3a, the first at $s = 0$ with an exponential distribution as above, and the second at $s \sim N$; i.e., now an avalanche can occupy the whole system. Thus, at low velocities, when the time dependence of the static frictional force is important, the slipping of junctions becomes synchronized. This can be explained qualitatively as follows. As was shown by

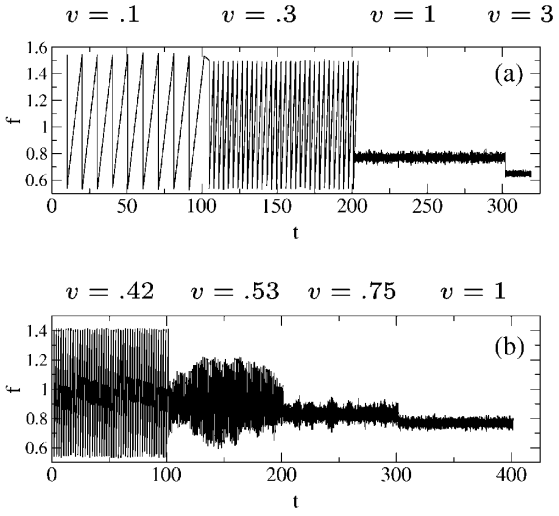

FIG. 2. (a) Total frictional force $f(t)$ vs time for velocities $v = 0.1$, 0.3, 1, and 3. $N = 30 \times 34$, $f_b = 0.1$, $f_s = 1$, $f_{sm} = 2$, $k_{int} = 0.1$, $\Delta r = 0.3$, and $\Delta x_{ini} = 1$. (b) Details of the transition for velocities around v_c: $v = 0.42$, 0.53, 0.75, and 1.

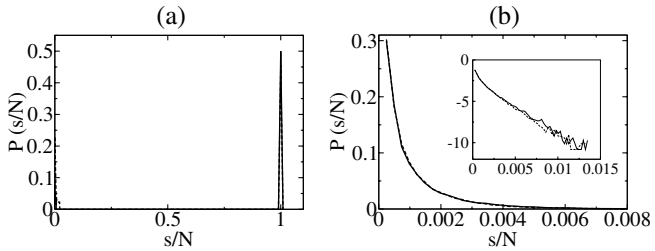

FIG. 3. The distribution of avalanche sizes $P(s/N)$ at (a) $v = 0.1$ (solid curve) and 0.2 (dotted curve); (b) $v = 3$ (solid curve) and 5 (dotted curve). The inset in (b) is a log-linear plot showing the exponential dependence. All data is for PBC with $N = 60 \times 68$, and the same parameters as in Fig. 2.

Persson [14], the distribution of forces $\mathcal{P}(f_i)$ now has a more complicated form; it is constant for forces $0 < f_i < f_s$ (recall $f_b = 0$) and monotonically decreases to zero for forces $f_s < f_i < f_{\rm sm}$. For purposes of an estimation let us assume that this decrease may be described by a simple linear dependence, $\mathcal{P}(f_i) = 2(f_{\rm sm} - f_i)/(f_{\rm sm}^2 - f_s^2)$. The condition that after the relaxation of the ith junction, the nearest neighboring junction jth will relax too, now takes the form $f_j(t - 0) \geq f_{si}(\tau_j) - bf_{si}(\tau_i)$. The probability that the jth junction will relax is equal to $p = \int_{f'}^{f_{\rm sm}} df\, \mathcal{P}(f) = (f_{\rm sm} - f')^2/(f_{\rm sm}^2 - f_s^2)$, where $f' = \langle (1 - b) f_{si}(\tau_i) \rangle$. With the parameters used above, if we take $\langle f_{si}(\tau_i) \rangle \approx 0.5(f_{\rm sm} + f_s)$, we obtain $p > \frac{1}{7}$. Moreover, if $\langle f_{si}(\tau_i) \rangle < 1.43$, we obtain $p > \frac{1}{5}$, so that $\nu p > 1$, and the avalanche will expand over the whole system.

Finally, *the transition is smooth* (see Fig. 2b); it is neither discontinuous (first order) nor continuous (second order) contrary to predictions of the phenomenological models [4].

Thus, the proposed variant of the earthquakelike model, which combines features of the OFC model and the phenomenological one, resolves the long-standing disagreement between experimentally observed $v_c \sim 1$ μm/s and MD results of $v_c \sim 1$ m/s. A single junction itself has to behave according to MD predictions; it should exhibit hysteresis and atomic-scale stick-slip motion. *The experimentally observed smooth sliding corresponds to atomic-scale stick-slip motion of many junctions.* This statement is in agreement with recent experimental results [21]. *The macroscopic-scale stick-slip behavior emerges due to the concerted motion of the many junctions due to their interaction, and the transition itself is smooth.* This prediction should be checked experimentally: first, the fluctuations of f in the "smooth" sliding regime should scale as $N^{-1/2} \propto 1/\sqrt{A}$ with the total area A of the contact, and second, a careful analysis of $f(t)$ should show a continuous spectrum for atomic-scale stick slip, while for steady sliding the spectrum should exhibit characteristic peaks at the washboard frequencies [7,14].

Two questions still remain unclear. First, the nature of the junctions was not specified in the model. They may correspond to real asperities, or to "solid islands" in the fluidized lubricant [14]. Second, the mechanism and precise $f_s(t)$ dependence remain unexplained. It may be due to the squeezing and aging of individual contacts, or due to the gradual increasing of the contact area, with the coalescing of smaller contacts into fewer larger contacts. All these processes involve plastic deformations and thus are characterized by macroscopic-scale times $\tau \sim 1$–10^3 s. Careful experiments and/or large-scale simulations of multicontact interfaces are needed to resolve these remaining questions.

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[20] For $v \ll 1$, our model again reduces to the OFC one with the threshold force $f_{\rm sm}$, and the stick-slip motion disappears. However, this is an artifact of the simple dependence (1). In a real system $f_s(t)$ has to continue to grow at $t \gg 1$ as well, although it may be slower and with another law. For example, if we take $f_s(t) = c_1 + c_2(t + c_3)^{1/2}$, where the parameters $c_{1,2,3}$ are adjusted to show a behavior similar to the dependence (1) at short times, then we get stick slip for any $v \ll 1$. Also, at $t \gg 1$ the number of junctions has to decrease due to the coalescence of nearest junctions, resulting in stick-slip motion, too.
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