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Kinetics of stick-slip friction in boundary lubrication

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Abstract – We show that new important features are brought to the kinetics and dynamics of frictional stick-slip motion in an earthquakelike model of boundary lubrication by introducing a distribution of static breaking thresholds of individual contacts. In particular the condition for elastic instability and details of the slip motion are heavily affected. Among the novel emerging properties is the role of other parameters such as the delay time of contact reforming, the strength of elastic interaction between the contacts, and the elasticity of the contacts and of the slider. We simulate the model dynamics, choosing parameters appropriate to describe a recent surface force apparatus experiment (Klein J., *Phys. Rev. Lett.*, **98** (2007) 056101) whose results are now explained with a totally normal boundary lubricant film viscosity.

Introduction. – This letter presents a theoretical model and related simulations of frictional stick-slip across an atomically thin, but macroscopically sized, film of lubricant ("boundary lubricant"). This time-honored problem has attracted over the years a large amount of experimental and theoretical work, as we will detail below. One problem in this area remains the need to access clear, accurate data suitable for a fully quantitative assessment of theoretical models. Luckily, in the surface force balance (SFB) technique [1–3], lubricated friction with lubricant films down to atomic thicknesses have nowadays become measurable with high accuracy between macroscopically sized ($\sim 20\,\mu$m) atomically smooth mica surfaces. In the experiment, friction exhibits a clear stick-slip. The slip event, in the ms range, is noticeably slowed down by the lubricant relatively to a strictly inertial duration $\tau_s \sim \Omega_S^{-1} \sim 10^{-3}$ s (here $\Omega_S = (K/M)^{1/2}$ is the natural frequency of the slider, of mass M and spring constant K). If, as is sometimes assumed, the slip is accompanied by a uniform, massive lubricant melting event (boldly extending to macroscopic sizes the nanosize melting suggested by early simulations [4–6]) then the delaying factor would be the lubricant's viscosity. In turn that would require viscosity values 10^4 to 10^7 times higher than the bulk lubricant viscosity [3,7,8]. The latter, however, is in conflict with squeezing experiments [9,10].

Our observation is that a thin lubricant film will more realistically be inhomogeneous at the mesoscale lengths involved in these experiments, so that the description of slip as a single massive lubricant melting event following depinning must be, even for ideally flat sliding surfaces, an oversimplification. On a mesoscopic or macroscopic length scale the atomically thin solid lubricant film will form domains with different orientation, possibly different structure and generally different yield stress thresholds [4]. This inhomogeneous film is unlikely to melt massively and to begin sliding all at once; rather, different domains will generally yield and start sliding one by one, as in the well-known earthquake (EQ) Burridge-Knopoff models, largely employed in a great variety of frictional studies [11–18]. Our scope here is to work out an implementation of the model suited to describe the onset of stick-slip in the SFB boundary lubrication experiments, and to explain the experimental results in this realistic frame.

Persson [14] first adjusted the EQ model to describe ordinary friction and to describe qualitatively both the stick-slip and the smooth-sliding regimes observed in laboratory experiments. Being one-dimensional, Persson's

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model cannot of course be expected to reproduce completely and quantitatively the full SFB experimental behavior. An extension of the model to two dimensions [15] improved considerably the agreement with experiment, including a prediction of the critical velocity of the transition from stick-slip to smooth sliding. Recently [17,18] it became clear that a crucial ingredient of the EQ-like model in tribological applications is the distribution of static yield thresholds for the breaking of individual contacts. In the early variants of the model, all contacts had been assumed to be identical for the sake of simplicity, and a distribution of thresholds appeared only implicitly due to temperature fluctuations [14] or due to interaction between the contacts [15]. As a matter of fact, the EQ model with identical contacts is a singular case [17]. It admits a periodic solution which can be interpreted as a form of stick-slip. Nonetheless this solution remains largely unphysical since for example it ceases to exist as soon as nonequivalent contacts are considered, whatever their precise properties. As soon as a finite width distribution of yield thresholds is taken into account, the solution of the model in the quasistatic limit always approaches a physical solution with smooth sliding [17,18]. Incorporating at the outset a threshold distribution allows to find the steady-state solution of the EQ model analytically, and more importantly to find conditions for appearance of the elastic instability, which is the necessary condition for the stick-slip to emerge [18,19]. Work along this line [18] concentrated so far mainly on general aspects and on identifying a steady-state solution of the EQ model with the threshold distribution.

Here we study in detail the *kinetics* of the model, with parameters addressing the specific case of experimental interest. By doing that, we find that a realistic EQ model of boundary lubrication can explain in great detail all frictional features observed in SFB experiments and particularly the observed slow slip dynamics. The gradual yielding of contacts one after the other is the factor slowing down the onset of slip, and making it slower than inertial, quite independently of the lubricant's viscosity. A simple change of experimental parameters can easily test this conclusion. In this study we find that other important parameters controlling the occurrence of stick-slip against smooth sliding, as well as details of the slip, include the width of the distribution of contact breaking thresholds (responsible for the elastic instability) and a finite delay time for contacts reforming which is the second necessary condition for stick-slip to occur. Altogether, our treatment underlines the necessity to include the inhomogeneity and complexity of real interfaces in the description of all macroscopic and mesoscopic stick-slip including SFB.

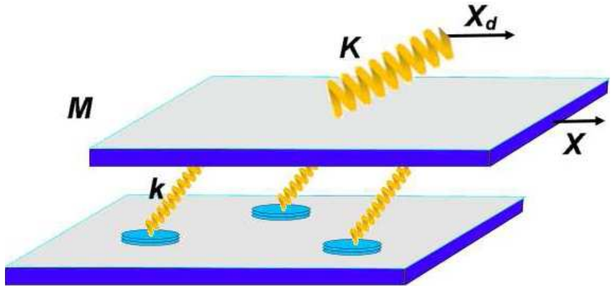

Fig. 1: (Colour on-line) The earthquakelike model.

Model. – We use a variant of the Burridge-Knopoff spring-block model of earthquakes adapted to laboratory tribology problems by Persson [14]. The model consists of a planar array of contacts (representing, *e.g.*, patches of lubricant, or 2D crystalline domains) $i = 1, 2, \ldots, N$, each connecting the base and the slider through a spring of elastic constants k_i (fig. 1). At rest and zero external stress, these N contacts are assumed to form a perturbed 2D lattice, where positions $\mathbf{r}_i^{(0)} = (x_i^{(0)}, y_i^{(0)})$ are obtained by shifting triangular lattice positions of spacing a by random Gaussian shifts Δx_i, Δy_i of zero mean and width Δx. Under shear, the contacts move to $\mathbf{r}_i = \mathbf{r}_i^{(0)} + \mathbf{l}_i$, where $\mathbf{l}_i$ are stretchings (relative to the base), and the lateral force from the base on the i-th contact is $\mathbf{f}_i^{(\mathrm{sub})} = -k_i \mathbf{l}_i$. As known in surface physics [20–23], the energy of elastic interaction between local defects on a surface of semi-infinite crystal falls off with distance as $\propto r^{-3}$. Therefore, we assume that contacts also interact elastically through pairwise forces $\mathbf{f}_{ij}^{(\mathrm{int})} \approx 3\, g(\mathbf{r}_i - \mathbf{r}_j)/(r_{ij}^0)^5$, where g is the strength of interaction. The total force exerted on contact i by the base and by other contacts is thus $\mathbf{f}_i = \mathbf{f}_i^{(\mathrm{sub})} + \sum_j \mathbf{f}_{ij}^{(\mathrm{int})}$. In addition, we assume each contact to be coupled "frictionally" to the slider, as follows. The contact moves rigidly with the slider so long as the total force $|\mathbf{f}_i| < f_{si}$, where f_{si} is an upper threshold value. Above that threshold the contact detaches from the slider and slips relative to it for a time τ_i after which it stops and attaches again to the slider. During slip, the contact feels a drag force from the slider $\mathbf{f}_i^{(\mathrm{drag})} = m_i \eta_i (\mathbf{v}_s - \dot{\mathbf{l}}_i)$, where $\mathbf{v}_s(t)$ is the slider velocity, m_i is the mass of the contact and η_i a local viscosity parameter of the lubricant. After slip, the contact sticks, re-attaching to the slider. Stick occurs when the total force $|\mathbf{f}_i|$ drops below a lower threshold value $f_{bi} \ll f_{si}$, and with a delay time τ_i. We will assume contact independent values η, τ_d and f_b for η_i, τ_i and f_{bi}, respectively.

Three parameters f_{si}, m_i, and k_i are thus associated with each contact. We assume that the thresholds f_{si} take random values from a Gaussian distribution of mean value f_s and standard deviation Δf_s. The continuous distribution of thresholds is the main factor affecting the initial slowing-down of the slip, to be discussed later. Both f_{si} and m_i should be proportional to the contact area $A_i = \pi a_i^2$ (a_i is the contact radius), while the contact stiffness is given by $k_i \approx \rho\, c^2 a_i$ [14], where ρ is the mass density and c is the transverse sound velocity of the material which forms the contacts (*i.e.*, of the substrate or the lubricant). Therefore, we take $m_i = m f_{si}/f_s$ and $k_i = k\,(f_{si}/f_s)^{1/2}$, where m and k are the mean mass and stiffness of the contacts. When a contact is "reborn" its parameters are assigned new values.

The slider is modelled as a rigid body of mass M and center-of-mass coordinate $\mathbf{R} = (X, Y)$, all elastic and plastic phenomena restricted to the contacts. The slider is subject to an external force exerted through a spring of elastic constant K moving with speed v_d relative to the base. In addition, the slider experiences the force $\mathbf{F} = \sum_i(\mathbf{f}_i + \mathbf{f}_i^{(\mathrm{drag})})$ from all contacts (either pinned or sliding). Its motion is thus described by

$$M\ddot{X} + M\eta_S\left(\dot{X} - \langle\dot{X}(t)\rangle\right) = F_x(t) + K(X_d - X), \quad (1)$$

where $X_d = v_d t$ and $\langle\dot{X}(t)\rangle = \eta_S \int_{-\infty}^{t} \mathrm{d}t'\, \dot{X}(t')\, e^{-\eta_S(t-t')}$. Here, inertial oscillations of the slider at its natural frequency Ω_S are attenuated through a damping coefficient η_S. It should be emphasized that this damping acts only on the motion of a part of the slider relative the other parts, thus mimicking internal degrees of freedom of the slider not explicitly taken into consideration, and does not influence the overall motion of the slider with a constant velocity. Finally, similar equations apply for the Y coordinate, with $Y_d \equiv 0$.

Parameters. – For the sake of quantitative results, we choose to model the freshest and highest-resolution SFB experiment whose description [3] fortunately permits almost all model parameters to be extracted. The slider mass and the setup stiffness are $M = 1.47\,\mathrm{g}$ and $K = 97\,\mathrm{N/m}$, so that $\Omega_S = 257\,\mathrm{s}^{-1}$ and the "natural" period $\tau_S = 2\pi/\Omega_S = 0.0245\,\mathrm{s}$. For the slider inner damping we use $\eta_S = 0.2\,\Omega_S$ which reproduces the experimentally observed attenuation of ringing oscillations [3]. We assume a four-layer OMCTS lubricant film of thickness $h = 3.5\,\mathrm{nm}$, and contact area $A = 10^{-10}\,\mathrm{m}^2$ [2]. Accordingly, we have $a = (A/N)^{1/2}$ for the average distance between the contacts, $A_i = A/N$ for the average area of a contact, $a_i = (A_i/\pi)^{1/2}$ for its average radius, and $m = \rho_{\mathrm{OMCTS}} A_i h$ for its average mass. For the contact viscosity we use $\eta = 2 \times 10^{11}\,\mathrm{s}^{-1}$, obtained from the bulk viscosity of the OMCTS lubricant, $\widetilde{\eta}_{\mathrm{OMCTS}} \approx 2.5 \times 10^{-3}$ Pa·s (at room temperature). The static threshold force value is taken as $f_s = F_s/N$ with $F_s = 18\,\mu\mathrm{N}$ [2,3]. For the remaining parameters, we used $kN = 2000\,\mathrm{N/m}$, $\Delta f_s = 0.01 f_s$, $f_b = 0.1\, f_s$, $v_d = 0.1\,\mu\mathrm{m/s}$, and $\tau_d = 5 \times 10^{-4}\,\mathrm{s}$, but we checked dependence of the results upon their changes. Finally, we take $N = 60 \times 68 = 4080$ and $\Delta x_i = 0.03\,a$ for contact number and geometry.

Results. – Numerical solution of the EQ model can be found either with the cellular automaton algorithm [13,15], or by solution of the master equation [18]; then eq. (1) is solved by standard Runge-Kutta method. Figure 2 shows the calculated time dependent spring force $F(t) = K(v_d t - X(t))$. The calculated frictional dynamics is strikingly similar to that observed in experiment, down to details

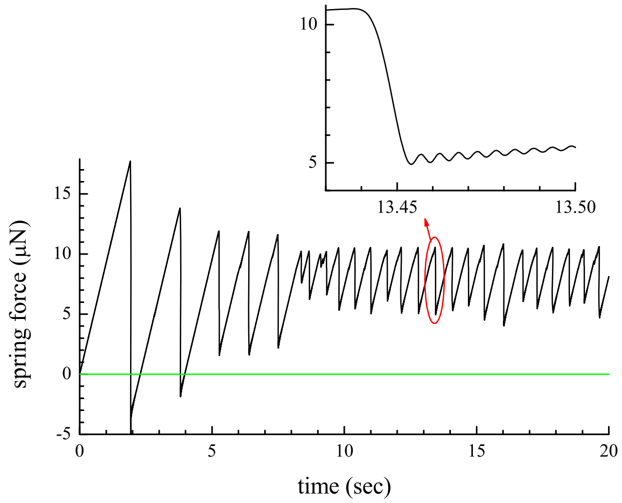

Fig. 2: (Colour on-line) Calculated slider spring force in the stick-slip regime. Parameters as given in the text, and $g = 0$. The inset shows the detail of a slip, with the sudden force drop and mechanical ringing oscillations.

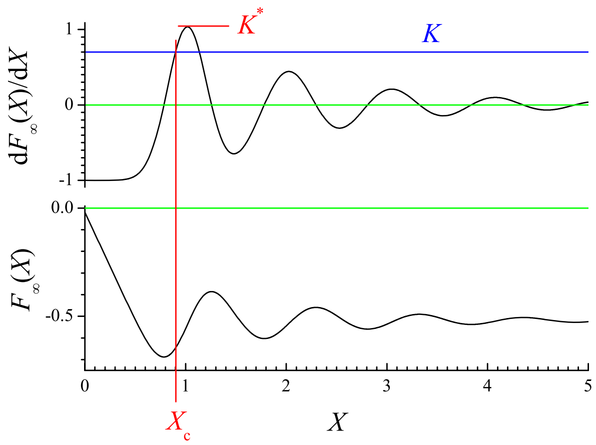

Fig. 3: (Colour on-line) Force *vs.* displacement of the rigidly coupled slider ($K = \infty$) with $\Delta f_s/f_s = 0.2$ and $Nk = 1$. The oscillations are due to the alternate prevailing of contact breaking ($|F_\infty|$ drops) and contact reforming (rises). Their damping leads finally to smooth sliding.

such as the large initial stick force spike, and the smaller amplitude of subsequent stick spikes (compare fig. 2 with figs. 1 and 2 in ref. [3]). More generally, the model yields either stick-slip, as in fig. 2, or smooth sliding as in fig. 3, depending on parameters. Stick-slip takes place quite generally, unless a) the pulling spring is too stiff, $K > K^*$; b) the contact reattachment delay time τ_d is too short; c) the Gaussian spread of threshold forces Δf_s is excessive; d) the average slider velocity v_d is too large. These conclusions, the overall behavior of $F(t)$ and its dependence upon parameters are rationalized as follows.

a) *Role of spring stiffness.* Consider first a rigidly coupled slider, $K = \infty$. In this limit it was shown analytically in ref. [18] that when the slider begins to move adiabatically, $\dot{X} > 0$, it experiences from the interface a friction force $F_\infty(X) < 0$ typically shown as a function of position in fig. 3. The sliding is smooth (except for the singular case of $\Delta f_s = 0$), the force $F_\infty(X)$ attaining asymptotically a position independent value, but the oscillations represent the germ of stick-slip. Initially $|F_\infty|$ grows linearly with X up to $\sim N(f_s - \Delta f_s)$ as the contacts elongate. Gradually however contacts begin to break and reform, stopping the increase of $|F_\infty|$ and inverting the slope through a displacement $\Delta X \approx \Delta f_s/k$ until the force reaches $N f_b$, where almost all contacts have been reborn. This is reminiscent of a Volterra oscillation in a predator-prey problem. Similar to that case the process repeats itself with a smaller amplitude until, due to increasing dispersion of breaking and reforming processes, the force asymptotically levels off in smooth sliding. Smooth sliding persists for a nonrigid slider as well, so long as the pulling stiffness is large enough, $K > K^*$, where $K^* = \max F'_\infty(X)$, at least for v_d not too large so that the motion is adiabatic. When conversely the spring is soft enough, $K < K^*$, there is an elastic instability [19]. The sliding motion becomes unstable at X_c, where X_c is the (lowest) solution of $F'_\infty(X) = K$ (see fig. 3 and ref. [18]). Omitting the details, one finds that as $X(t)$ grows past X_c for $t > t_c$, the spring force quickly drops during an inertial slip time $\tau_s \approx \alpha \Omega_S^{-1}$ in the form $F(t) \approx F(X_c) - (K^2 v_d/6M)(t - t_c)^3$, where $\alpha = (6 F_s \Omega_S / K v_d)^{1/3}$ ($\alpha \approx 2.21$ for the chosen set of parameters)[1]. One may think that this instability will result in stick-slip motion. However, the condition $K < K^*$ is only necessary but not sufficient to produce stick-slip.

b) *Role of contact delay time.* The second necessary condition for stick-slip is a sufficient delay time $\tau_d > 0$ for contact reforming. If $\tau_d = 0$ the system will in fact still end up in smooth sliding, at least so long as the mechanical ringing vibrations can be ignored[2]. The friction force dependence upon τ_d is shown in fig. 4. When $\tau_d \ll \Omega_S^{-1}$, after the first few slips the spring force drop fails to reach below F_b, and eventually smooth sliding is reached, despite $K \ll K^*$ (fig. 4a). When $\tau_d < \Omega_S^{-1}$ the spring force grows after the slip, it rings with a frequency $\Omega_L = (Nk/M)^{1/2}$, but now stick-slip prevails (fig. 4b). When finally $\tau_d \geqslant \Omega_S^{-1}$, the spring force drops to negative values, it rings around zero for a time τ_d with the setup frequency Ω_S and stick-slip dominates as demonstrated in fig. 4c (in a real setup, the ringing frequency will also be determined by the mass and rigidity of the bottom substrate). The parameter τ_d

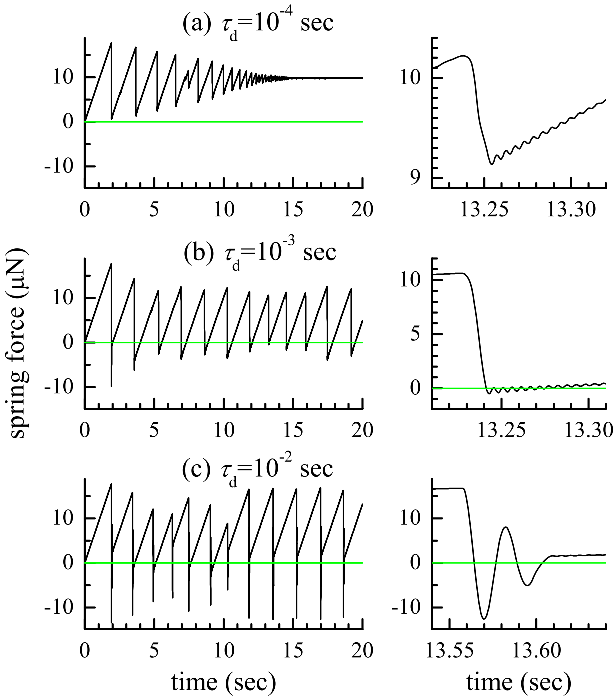

Fig. 4: (Colour on-line) Frictional force $F(t)$ for different values of the delay time τ_d. Despite the soft spring constant $K = 97\,\mathrm{N/m} \ll K^* \sim 10^5\,\mathrm{N/m}$, stick-slip is only found for sufficiently large τ_d.

is the time needed for formation of a new contact (bridge, asperity, etc.). For example, if the lubricant is locally melted because of sliding, τ_d corresponds to the time for nucleation and growth of a solid grain in the liquid lubricant[3].

c) *Role of dispersion parameter.* As we mentioned in Introduction, the dispersion Δf_s must be nonzero, otherwise the model has only the singular periodic solution, previously (and mistakenly) considered as stick-slip. The ratio $\Delta f_s / f_s$ controls the appearance of elastic instability. Therefore, the dispersion Δf_s of contact breaking thresholds should not be too large for stick-slip to ensue. We estimate that $K^* \approx Nk\,(f_s - f_b)/\Delta f_s$, so that if Δf_s rises, the sliding will be smooth unless the spring constant gets really small. In stick-slip, an increase of Δf_s leads to the decrease of the stick-slip period τ_{ss} as in fig. 5. The slip time duration τ_s also increases with Δf_s, but this effect is almost negligible.

d) *Role of sliding velocity.* If the sliding velocity is so small that the stick-slip cycle period τ_{ss} is large, $\tau_{ss} \gg \eta_S^{-1}$, then ringing is completely damped out during τ_{ss}. On the other hand when v_d is so high that $\tau_{ss} < \eta_S^{-1}$, then the contact reforming is progressively disturbed and that may lead to smooth sliding.

[1]Were we to assume, following [3], the instant melting of the whole lubricant film, then $F(t)$ would drop as $F(t) \approx F(X_c) - B(t - t_c)^2$ with $B = [F(X_c) - N\eta m v_d]\Omega_S^2/2$, so that $\tau_s \approx \beta \Omega_S^{-1}$ with $\beta = [2/(1-b)]^{1/2}$ and $b = \eta m v_d / f_s$ ($b \approx 3.7 \times 10^{-7}$ for the chosen set of parameters). In that assumption, only by using artificially large values for η, could one make τ_s large as in experiment.

[2]In Persson's simulation [14] of the EQ model, where stick-slip was demonstrated, the delay time τ_d corresponds to the contact sliding time necessary for its velocity to fall to zero. However, the sliding time is only one (relatively small) contribution to τ_d.

[3]This is a separate, rather complicated but important physical problem. Details of contact aging should be considered too. Addressing in detail these delicate physical questions lies however clearly outside of the scope of the present letter.

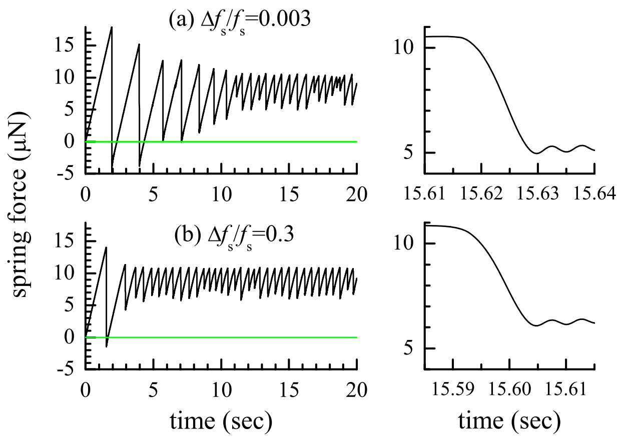

Fig. 5: (Colour on-line) Dependence $F(t)$ for different values of the dispersions Δf_s.

Details of other model parameters such as N, Δx, f_b, k, and η, are not essential, and while we have no space to show that, their variations do not change the essence of the results. In particular, the lubricant viscosity coefficient η may influence the results only if increased by more than four orders of magnitude, an increase which now there is no reason to believe. Increased interaction between the contacts is also uninfluential. Controlled by the dimensionless parameter $\xi = g/(f_s a^4)$, strong mutual contact interactions cause them to behave more concertedly, making stick-slip less irregular. Although the present model does not treat explicitly the aging of contacts [14,15], in some sense that is implicitly included through the delay time τ_d. For example, the ratio $\Delta f_s/f_s$ should decrease with stick time because of aging. Incomplete aging is involved in the decrease of stick-slip swing magnitude after the first few events; in the transition from stick-slip to smooth sliding with increasing driving velocity; and in other aspects, all of which could benefit from a more detailed treatment.

Discussion and conclusions. – In the present earthquakelike model of SFB boundary lubrication, the population of broken *vs.* unbroken contacts tends to oscillate spontaneously —a sort of Volterra oscillation. Stick-slip follows when simultaneously the slider is mechanically soft; when the contacts take enough time to be reformed; and when their static sliding thresholds are not identical but not too spread apart. The elastic energy stored in the slider during stick is partly dissipated during slip, and partly during the following ringing oscillations [3]. In our model the gradual breaking (melting) of different microscopic contacts (domains) is responsible for slowing down the detachment which initiates the slip. As a result, and contrary to previous claims, the increase of slip time τ_s over the bare setup inertial frequency Ω_S [24] is a slowing-down caused by the contact multiplicity, and not by the lubricant viscosity η, which influences the results very little. Our conclusion may easily be checked experimentally by changing, for example, the loading force. Indeed, the damping force associated with the lubricant viscosity in the uniform melting interpretation is directly proportional to the contact area, which in turn is roughly proportional to the load. We note nonetheless that lubricant parameters such as the dispersion $\Delta f_s/f_s$ and the delay time τ_d are rather important as they control the system dynamics. The mechanism and the model described here should have much broader applicability than just the example chosen. The great merit of boundary lubrication is however that the high-resolution SFB data have made the problem for the first time fully quantitative.

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