The exciton spectrum of the quantum dot - quantum ring semiconductor nanostructure in an electric field

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Motivation for research

Multilayer semiconductor nanostructures have been studied both theoretically and experimentally for quite a long time. The unique properties of quasiparticles in such systems make it possible to use them as basic elements in devices of modern nanoelectronics: tunnel nanodiodes, nanolasers, nanodetectors [1, 2]. A special place among various types of multilayer nanostructures is occupied by semiconductor quantum rings. As a rule, they have a cylindrical symmetry, as well as quantum wires, however, in contrast to them, their height is finite and amounts to a few nanometers. So the movement of charge carriers in such nanosystems is limited in all three dimensions. Therefore, in this aspect, they are similar to cylindrical quantum dots.

The aim of this work is to build one of the possible theoretical approaches to solving the problem of the exciton spectrum and the intensity of interband quantum transitions in in the quantum dot - quantum ring nanostructure in a constant electric field. The theory developed here makes it possible to analyze the dependence of the spectral parameters of the electron, hole and exciton on the magnitude of the strength *F*.

Theory of the exciton spectrum and intensities of interband quantum transitions in the quantum dot - quantum ring in an electric field

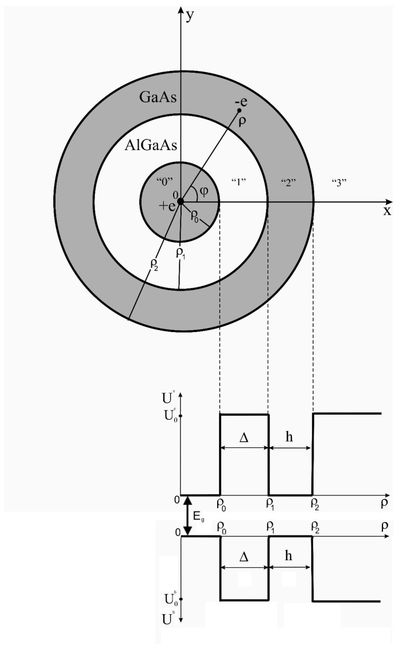

Fig.1. Cross section and energy scheme of the nanostructure

The paper investigates a nanostructure of height *L*, consisting of a cylindrical semiconductor cylindrical semiconductor quantum dot (quantum well, GaAs medium), which through a finite potential barrier ($Al_xGa_{1-x}As$ medium) is tunnel-connected to a coaxial cylindrical nanoring (quantum well, GaAs medium). The cross section and energy diagram of such a nanostructure is shown in Fig. 1. The electric fields strength vector $\vec{F}$ is directed along the *Ox* axis.

$$U(|\vec{r}_e-\vec{r}_h|)=-\frac{e^2}{\varepsilon(\vec{r}_e,\vec{r}_h)\,|\vec{r}_e-\vec{r}_h|} \qquad (1)$$

- the potential energy of the interaction of the electron and the hole in a medium with dielectric constant $\varepsilon(\vec{r}_e,\vec{r}_h)$, which in the general case is a complex function that depends on the spatial location of the electron and the hole in the nanosystem.

In order to investigate exciton states in such a nanosystem, it is necessary to solve the stationary Schrödinger equation with the Hamiltonian

$$\hat{H}_i(\vec{r}_i)=\frac{1}{2\mu^{(i)}(\rho_i)}\left[-\hbar^2\left(\frac{\partial^2}{\partial\rho_i^2}+\frac{1}{\rho_i}\frac{\partial}{\partial\rho_i}+\frac{1}{\rho_i^2}\frac{\partial^2}{\partial\varphi_i^2}\right)\right]-\frac{\hbar^2}{2\mu^{(i)}(\rho_i)}\frac{\partial^2}{\partial z_i^2}+U^{(i)}(\vec{r}_i)\mp|e|F\rho_i\cos\varphi_i\ ,\ (i=e,h) \qquad (2)$$

$U(\vec{r})$- dimensional quantization potential.

In the absence of electric field, the corresponding stationary Schrödinger equation is solved analytically exactly, and the wave functions are obtained in the form

$$F^{0}_{n_\rho m}(\rho,\varphi)=\frac{1}{\sqrt{2\pi}}R_{n_\rho m}(\rho)\,e^{im\varphi} \qquad (3)$$

$$f_{n_z}(z)=\sqrt{\frac{2}{L}}\begin{cases}\cos(\frac{\pi n_z}{L}z), & n_z=1,3,5,...\\ \sin(\frac{\pi n_z}{L}z), & n_z=2,4,6,...\end{cases} \qquad (4)$$

$$R^{(i)}_{n_\rho m}(\rho)=A^{(i)}_{n_\rho m}\,j^{(i)}_m(\chi\rho)+B^{(i)}_{n_\rho m}n^{(i)}_m(\chi\rho)\ ,\qquad (i=1,2,3,4) \qquad (5)$$

$$n^{(i)}_m(\chi\rho)=\begin{cases}K_m(\chi_0\rho), & i=0,2,4\\ N_m(\chi_1\rho), & i=1,3\end{cases}\qquad j^{(i)}_m(\chi\rho)=\begin{cases}I_m(\chi_0\rho), & i=0,2,4\\ J_m(\chi_1\rho), & i=1,3\end{cases}$$

$m=0,\pm1,\pm2,..$ - magnetic quantum number $n_\rho=1,2,3,...$ - radial quantum number

J_m, N_m - Bessel functions of integer order I_m, K_m - modified Bessel functions

All unknown coefficients in (5), as well as the electron energy spectrum, are found from the conditions of continuity of the wave functions and the corresponding fluxes of the probability densities at all heterointerfaces of the nanostructure and the normalization condition.

In order to solve the Schrödinger equation with the Hamiltonian (2) we write the unknown wave functions in the form of expansion of the complete set of wave functions (3)

$$F_n(\rho,\varphi)=\frac{1}{\sqrt{2\pi}}\sum_{n_\rho}\sum_m c^n_{n_\rho m}R_{n_\rho m}(\rho)\,e^{im\varphi} \qquad (6)$$

Substituting the expansion (6) into the Schrödinger equation with the Hamiltonian (2), we obtain the secular equation

$$\left|H_{n_\rho m,n'_\rho m'}-E_{n n_z}\delta_{n_\rho,n'_\rho}\delta_{m,m'}\right|=0$$

$$H_{n_\rho m,n'_\rho m'}=E_{n_\rho m}\delta_{n_\rho,n'_\rho}\delta_{m,m'}+\left(\delta_{m',m+1}+\delta_{m',m-1}\right)\frac{eF}{2}\int_0^\infty R_{n_\rho m}(\rho)\,R_{n'_\rho m'}(\rho)\,\rho^2\,d\rho \qquad (7)$$

The problem of finding the energy spectrum and wave functions is now reduced to the calculation of eigenvalues and eigenvectors of the resulting matrix.

The found energy spectrum and wave functions also make it possible to estimate the intensities of interband optical quantum electron transitions by a formula

$$I^{n^e n^e_z}_{n^h n^h_z}=\left|\int\psi^e_{n^e n^e_z}(\rho,\varphi,z)\,\psi^h_{n^h n^h_z}(\rho,\varphi,z)\,\rho\,d\rho\,d\varphi\,dz\right|^2 \qquad (8)$$

Analysis and discussion of results

The calculation and analysis of the electron, hole, and exciton spectra and the intensities of interband optical quantum transitions were performed by numerical methods on the example of a nanostructure based on $GaAs/Al_{0.4}Ga_{0.6}As$ semiconductors

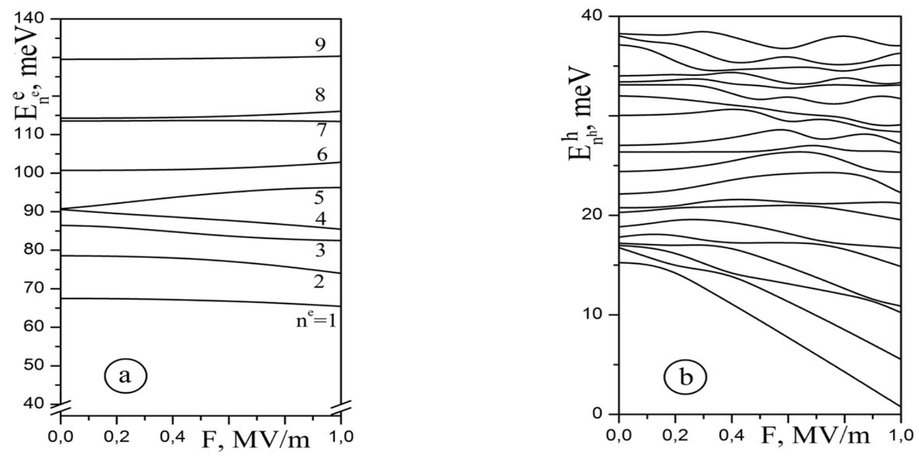

Fig. 2. Dependences of the energies of the electron $E^e_{n^e}$(a) and the hole $E^h_{n^h}$ (b) on the magnitude of the electric field strength *F* at $L=5\,nm,\ \rho_0=5a\ GaAs, h_1=18a\ GaAs, \Delta=30a\ GaAs, h_2=18a\ GaAs$

Figure 2 shows that the energy of the ground state of the electron and the hole only decreases with increasing *F*. However, this is no longer the case for excited states. In particular, the electron energy in the state with $n^e=5$ increases with increasing *F*. In the general case, the increase or decrease in the energies of an electron or a hole with increasing *F* is due to the quantum dot in which the electron is located in the corresponding states and the nature of the angular probability distribution relative to the direction of the electric field. Since the effective mass of the hole is almost an order of magnitude greater than the electron mass, the density of its energy levels in the energy scale of the quantum well is significant and pronounced anticrossings are observed in the corresponding dependences on F (Fig. 2b). The cause of anticrossing is a change in the localization of the quasiparticle between the inner quantum dot and outer quantum ring in adjacent quantum states with increasing electric field strength [3].

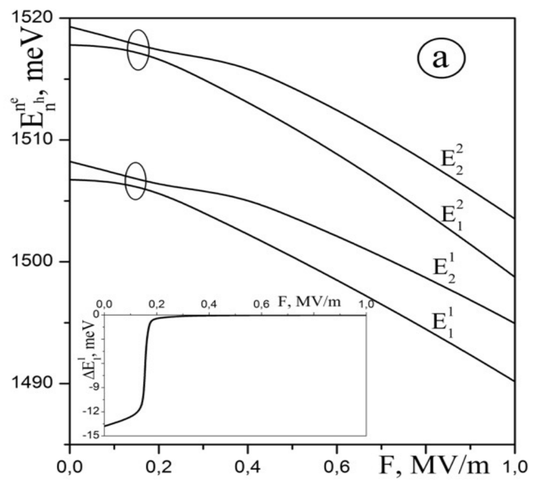

Fig. 3. The dependences of the energies of several lowest exciton levels $E^{n^e}_{n^h}$

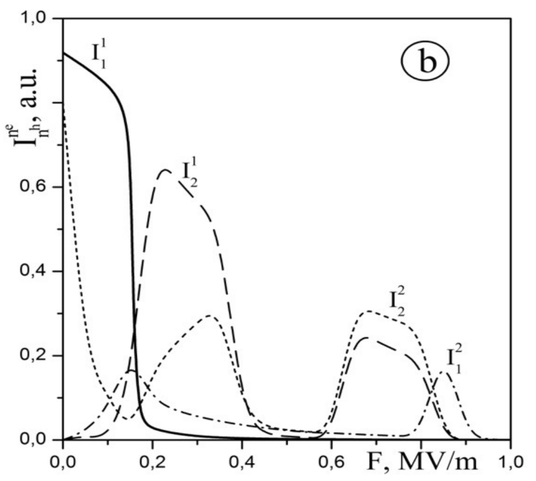

Fig. 4. The intensities of interband optical quantum transitions $I^{n^e}_{n^h}$ on the magnitude of the electric field strength (F) at $L=5\,nm,\ \rho_0=5a\ GaAs,$ $h_1=18a\ GaAs, \Delta=3a\ GaAs, h_2=17a\ GaAs$

The inset in Fig. 3 shows the dependence of the binding energy of the ground exciton state on F for the same geometric parameters of the nanosystem. The exciton binding energy is maximal in the absence of an electric field. It only decreases with increasing F, and at F = 0.5 MV/m is almost zero. This behavior of ΔE^1_1 is easy to understand by considering the dependence of the probability density of finding an electron and a hole in a nanosystem in the ground state $\left|F^{(e,h)}_1(\rho,\varphi)\right|^2\rho$ at *F = 0*, where ΔE^1_1 is maximum, and F = 0.5 MV/m, where it is minimal.

The dependences of the transitions intensities $I^{n^e}_{n^h}$ between other quantum states on *F* are nonmonotonic functions with pronounced maxima and minima (Fig.4). This behavior is due to the change in the hole localization between the inner quantum dot and outer quantum ring in excited states. The number of maxima is determined by the number of anticrossings of hole energy levels (Fig. 2b).

Note that the binding energy of the exciton is much less than the sum of the size-quantized energies of the electron and the hole. Therefore, the dependence of the energies of the exciton states on the strength F (Fig. 4) is mainly due to the peculiarity of the behavior of the energies of the electron and the hole. In particular, in these dependences exciton anticrossing is observed as a manifestation of anticrossing of hole energy levels.

Conclusions

The dependence of the energy spectrum of an electron, a hole, an exciton, and the intensities of interband quantum transitions on the magnitude of the electric field strength (*F*) in in the quantum dot - quantum ring based on $GaAs/Al_{0.4}Ga_{0.6}As$ semiconductors was investigated in the model of effective masses and rectangular potentials.

To calculate the energy spectrum and distributions of the probability density of finding an electron and a hole in the quantum dot - quantum ring in an electric field, the stationary Schrödinger equation is solved by decomposing an unknown wave function by wave functions of a quasiparticle in a nanosystem in the absence of an external field.

The energies of the hole, the exciton, and the intensity of the interband quantum transitions depend nonmonotonically on the magnitude of the electric field strength. In particular, the energy dependences on *F,* anticrossings of energy levels are observed, and in the dependences of the oscillator forces on *F* (except for the ground exciton state) pronounced maxima and minima are observed. The reason for this behavior is a change in the localization of the electron and the hole in the space of the quantum dot - quantum ring in different quantum states with a change in the electric field strength.

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