

Transition from order to chaos of incommensurate superstructure described by the Lifshitz invariance under conditions $n = 3$



Sveleba S.A.¹, Katerynychuk I.M.¹, Kuno I.M.¹, Velgosh A.S.¹,
Shmygelsky J.A.¹, Semotyuk O.V.², Kuno V.M.³

¹National Ivan Franko University of Lviv 107 Tarnavskogo st., Ukraine

E-mail: incomlviv@gmail.com

²Ukrainian Academy of Printing of Lviv 19 Pid Goloskom st., Ukraine

³Lviv Politechnic National University 12 St. Bandera st., Ukraine

The goal of this work is to construct bifurcation diagrams that illustrate the occurrence of chaos by the method of doubling the period for ferroelectric crystals with multiplication of the unit cell $n = 3$. To do this, we calculated the amplitude function of an incommensurate (IC) superstructure described by two second-order differential equations. This system of two second-order differential equations was solved by the numerical BDF method. The calculations were performed in the Python software environment, using the JiTCODE library.

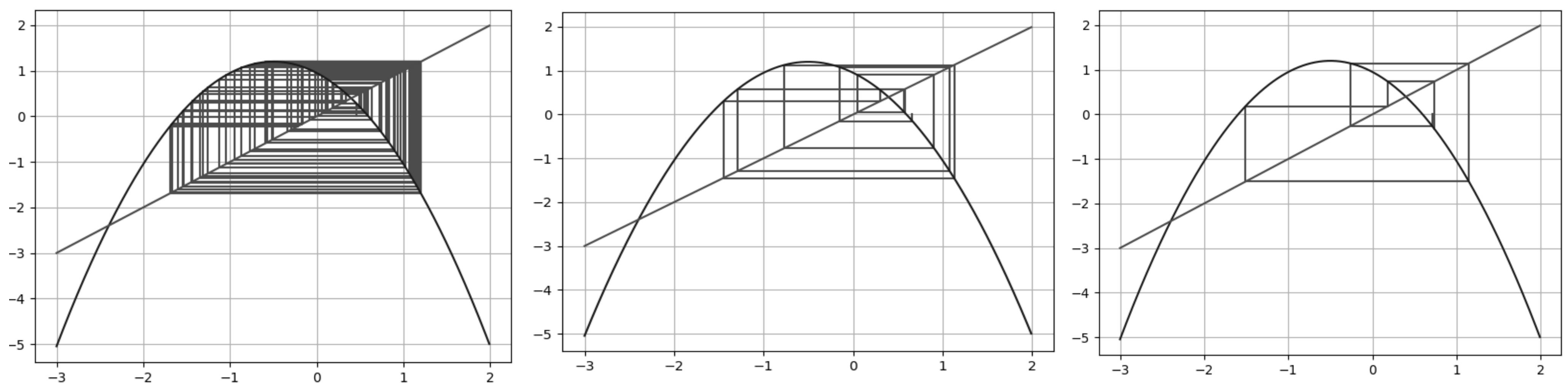


Fig.1. Diagram of the logistic function under influence of the surface energy of E .

Based on the studies of the branching diagram and the mapping function diagram for IC superstructure occurring in ferroelastic crystals with elementary cell multiplication $n = 3$, it can be stated that due to frequency doubling bifurcations the system goes into a chaotic state. The parameters T and K , which are responsible for long-range and anisotropic interactions, respectively, in the first approximation well describe the behavior of IC modulation, and its modes. It is established that at small values of long - range interaction ($T < 0.1$) there is an undeveloped chaotic state which is characterized by blurred processes of frequency doubling and their breaks. According to the diagrams of logistics functions, the influence of surface energy on IC superstructure causes an increase in the magnitude of the anisotropic interaction, which leads to the removal of the degeneracy of the system *Fig.1*.