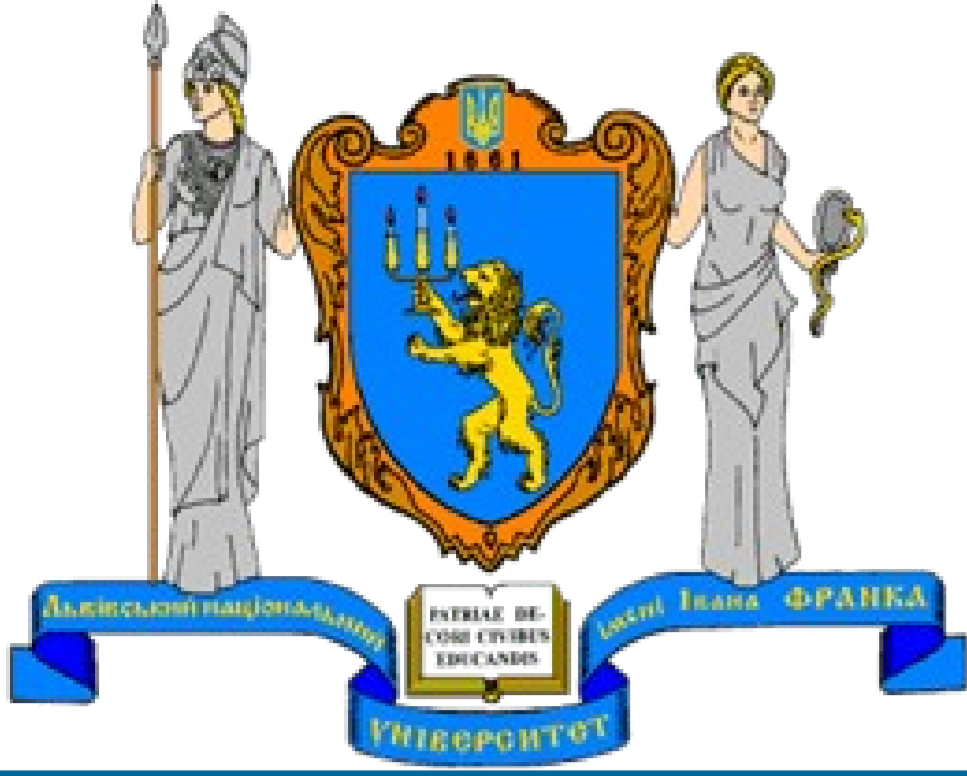# Temperature behavior of fractional power dispersive systems with medium dispersion

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Universal characteristics of medium dispersive systems

It is known that dielectric spectra of solids in low and radio-frequency range often obey power frequency law $\omega^{-(1-n)}$ with fractional exponent $0 < n < 1$ [1, 2]. Fractional power frequency dependencies of spectra it is not easy to explain. Particularly, a raw of models are proposed fo explanation of the frequency dependence of conductivity and dielectric permittivity and loss with $n \approx 0.8$ on the basis of different physical processes [3–8]. According to them a power frequency dependence is the consequence of either power frequency dependence of hopping time caused by overlapping of potentials of randomly distributed localized charge carriers as for Elliott model [4] or energy distribution of localized levels near Fermi level (Austin-Mott model [3]) or in the tails of allowed band energies. The models predicts different types of temperature dependence of dielectric constant and dielectric loss hardly confirmed experimentally. However, dielectric spectra mainly measured on amorphous materials show another type of temperature dependence, often showing some universality that should be expected from universality of frequency response.

Main universal features of the dielectric or admittance spectra with $n \approx 0.8$:

1. Weak super-linear increase of dielectric constant and dielectric loss with increasing temperature that may be fit by exponential temperature dependence $\exp(T/T_0)$, known as inverse Arrhenius law.

2. At lower frequencies temperature dependence of dielectric constant and loss are stronger showing lower value of parameter T_0.

3. Often the exponent n decreases with increasing temperature according to a linear law.

4. The temperature dependencies of dielectric constant and loss is weaker than for an usual thermally activated Arrhenius processes.

5. Very weak temperature dependencies of dielectric constant and loss are observed for spectra with n approaching 1.

6. Temperature dependencies of dielectric loss or acconductivity are frequency dependent.

7. Limitation of range of the values of complex dielectric constants and complex ac conductivity measured experimentally for dielectric response with $n \approx 0.8$.

Let us consider the system with power fractional distribution of dipoles on their relaxation time.

$$g = \frac{N(f, f + \Delta f)}{\Delta f} = \frac{g_0}{\omega^{(2-n)}} \qquad (1)$$

where g_0 is constant, ω_0 is circular frequency. To obtain the frequency dependencies of $C_1(\omega)$ and $C_2(\omega)$ of the system where effective dipoles are distributed according to Eq. (1) the following integral should be calculated

$$C^*(\omega) = C_1(\omega) - jC_2(\omega) = \int_0^\infty \frac{g(\omega_0)}{(1 + j(\omega/\omega_0))}\, d\omega_0 =$$
$$= \int_0^\infty \frac{g_0 \omega_0^{-(1+n)}}{(1 + j(\omega/\omega_0))}\, d\omega_0 \quad (2)$$

We obtain that both $C_1(\omega)$ and $C_2(\omega)$ depend on frequency according to the same fractional power law with exponent $-(1-n)$, but with different multiplying factors depending on n.

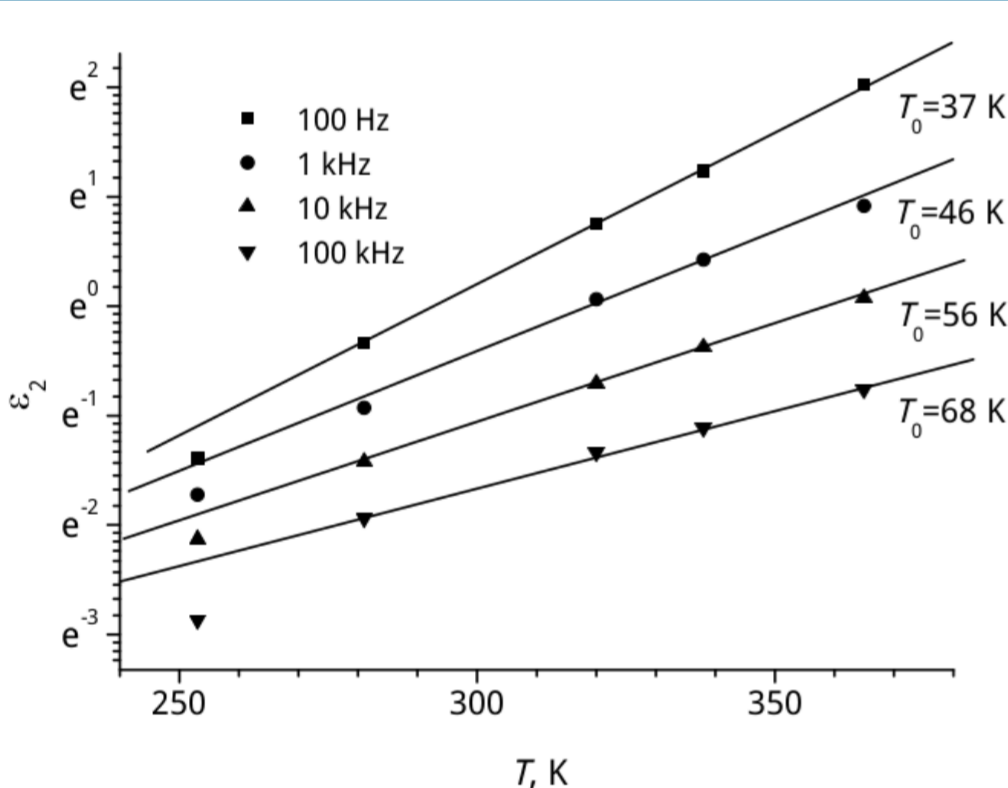## Some experimental temperature dependencies of materials with medium dispersion systems

FIG. 4: Temperature dependence of dielectric loss for glass transition metal oxide 50P$_2$O$_5$–10BaO–40FeO on different frequencies (1 - 100 Hz, 2 - 1 kHz, 3 - 10 kHz, 4 - 100 kHz) (from fig. of [15]).
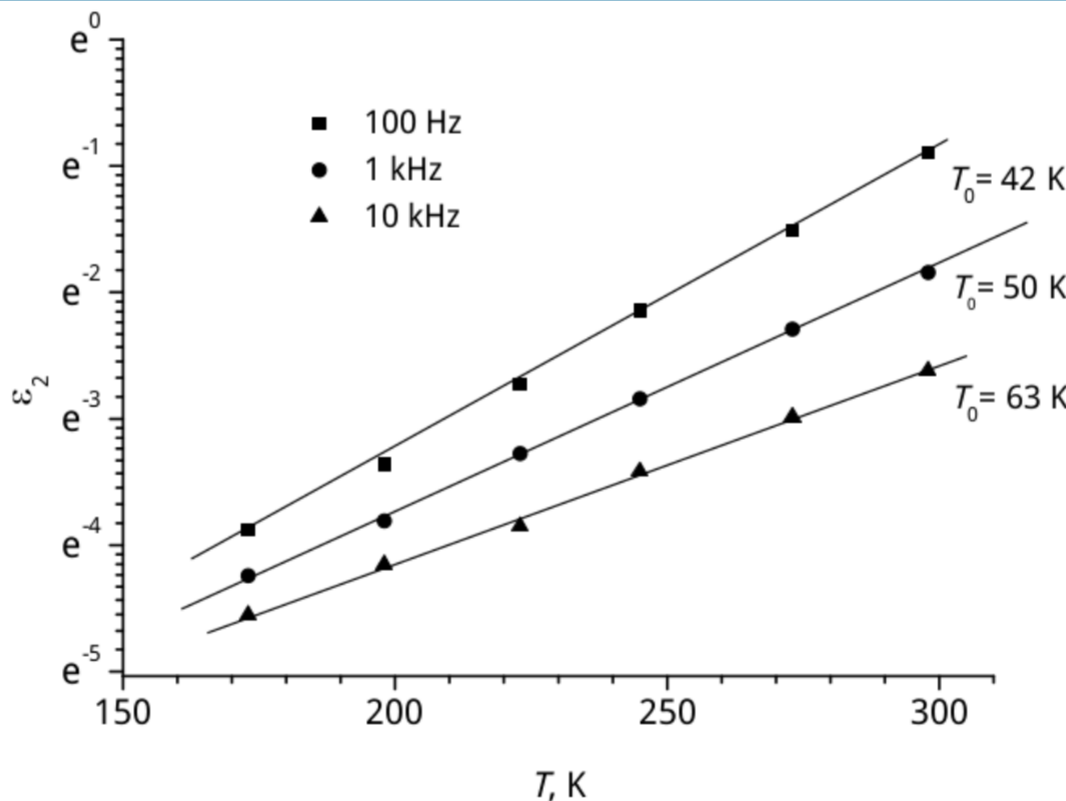

FIG. 5: Temperature dependence of dielectric loss on different frequencies for evaporated SiO film (sample S51) (1 - 100 Hz, 2 - 1 kHz, 3 - 10 kHz) (digitized from fig. 4 of [14]).
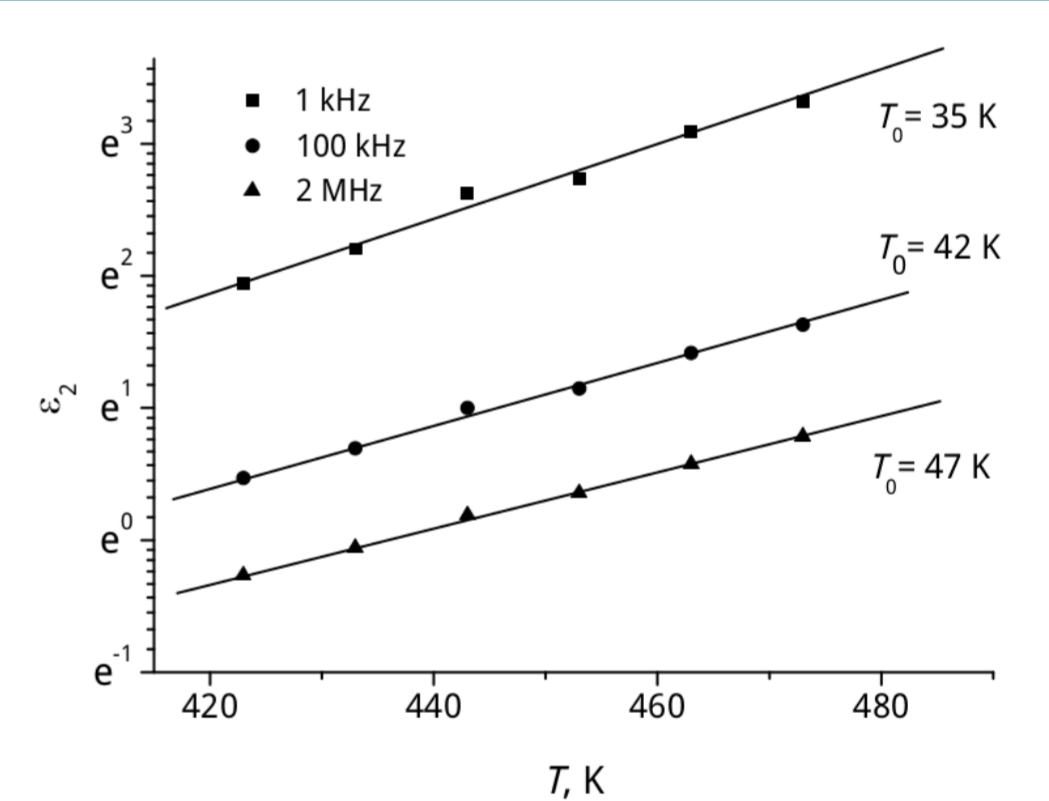

FIG. 6: Temperature dependence of dielectric loss on different frequencies for 40ZrF$_4$–20BaF$_2$–10YF–(30-x)LiF–xNaF with $x = 15$ glass (1 – 1 kHz, 2 – 100 kHz, 3 – 2 MHz) (digitized from fig. 1 of [16]).

Characteristics of proposed model of temperature behaviour

Calculate the magnitude of distribution on frequency f considering the distribution function of effective dipoles on log-log scale assuming the power frequency dependence of g on characteristic frequency ω_0. On frequency $f - df$ the relative increase of dg/g should be on $(1 - n)$ factor less then df/f shift.

$$\frac{dg}{g(\omega)} = (1-n)\frac{df}{f} = (1-n)\, ln\left(\frac{f_0}{f}\right)\frac{dT}{T} \qquad (6)$$

Increasing of dg/g is stronger on lower frequencies that leads to increasing of $(1 - n)$ and slope of the effective dipoles distribution on log-log scale with increasing temperature. Solving the equation we obtain that $g = g_0\ \exp(T/T_0)$ where $T_0 = (1-n)\ln\left(\frac{f_0}{f}\right)$. According to (1) it could be state that temperature dependence of complex capacity has the same shape. Increasing of $g(\omega_0)$ with increasing temperature consists of two parts: frequency independent and frequency dependent ones (Eq. (6)). Since frequency dependent term is proportional to $ln(f)$, the shape of dipoles distribution within the frame of the proposed model stay obeying to fractional power law with changing of temperature.

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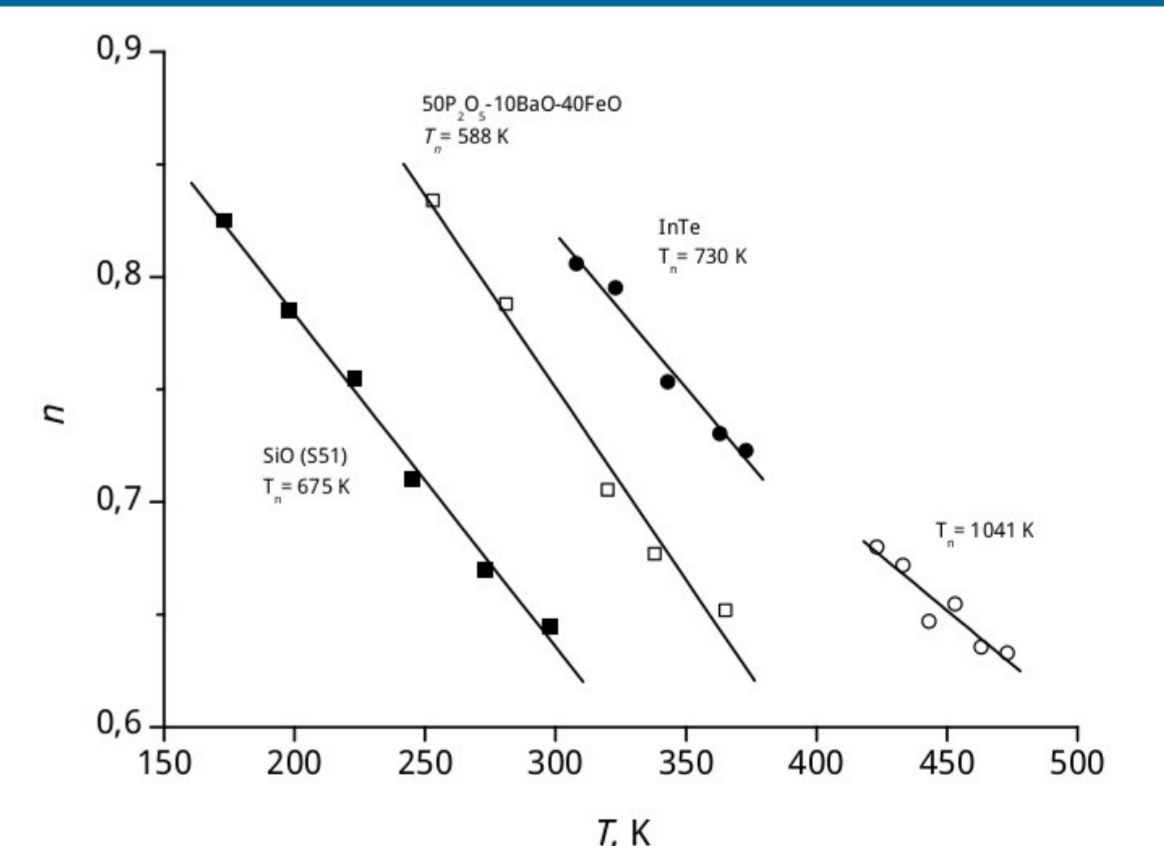

FIG. 3: Temperature dependence of power n for different materials (1 - SiO (S51) ($T_n = 830$ K) [14], 2 - 50P$_2$O$_5$–10BaO–40FeO ($T_n = 588$ K) [15], 3 - 40ZrF$_4$–20BaF$_2$–10YF$_3$-xLiF-xNaF ($T_n = 1042$ K) [16], 4 - InTe single crystal ($T_n = 730$ K)

The solution of this equation may be written in the shape

$$C_2 = A \exp\left(\frac{T}{T_0}\right) = A \exp\left\{ ln\left(\frac{f_0}{f}\right)\frac{T}{T_n} \right\} \qquad (7)$$

Conclusion

Now, it is difficult to declare, that all dispersive dielectric systems characterizing by $n \approx 0.8$ obey inverse Arrhenius law with changing a temperature. Since spectra with $n \approx 0.8$ obey weak temperature dependence for reliable confirmation it is necessary to have many dielectric spectra dominating by clear response with $n \approx 0.8$ within as more as possible wide frequency range without any additive polarization processes.

However, proposed universal model of temperature behavior of spectra with $n \approx 0.8$ with high probability may be valid for wide range of the disordered materials, that is confirmed by universal applicability of fractional power frequency dielectric response in solid state physics.