Self-affine dynamics of ice surface softening at friction

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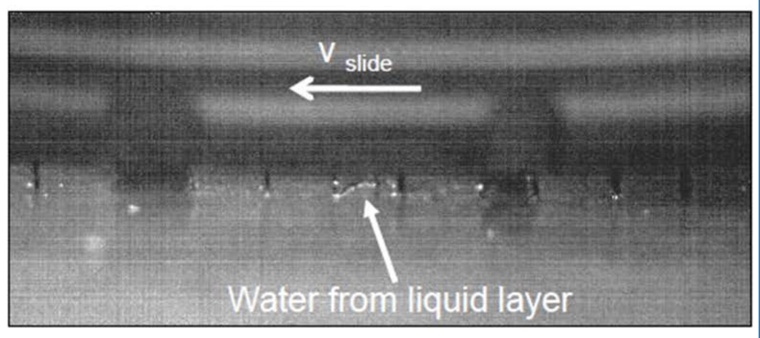
Abstract

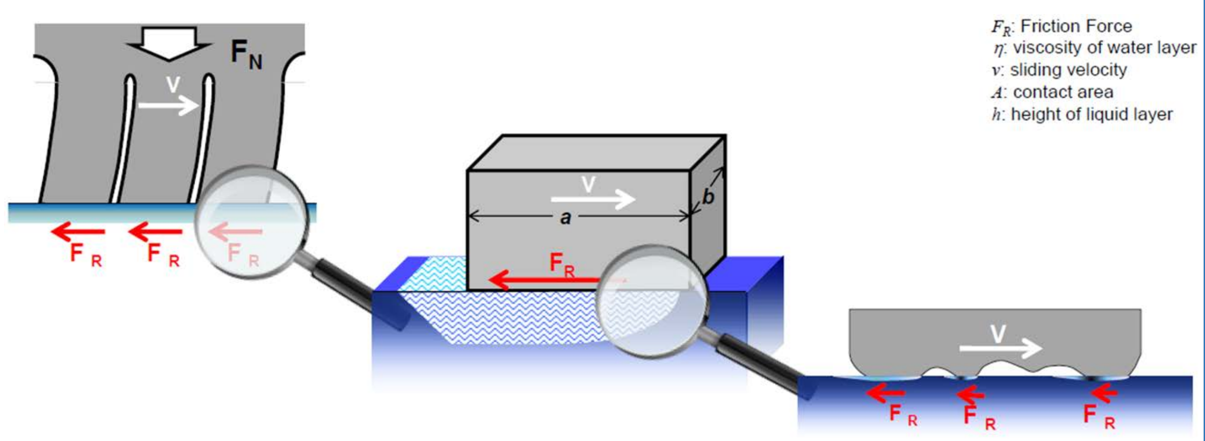
Softening of ice surface under friction is explored in terms of the rheological model for viscoelastic matter approximation. The nonlinear relaxation of strain and fractional feedbacks are allowed. Additive non-correlated noise associated with shear strain, stress as well as with temperature of ice surface layer, is introduced, and a phase diagram is built where the noises intensities of the stress and temperature define the domains of crystalline ice, softened ice, and two types of their mixture (stick-slip friction). Conditions are revealed under which crystalline ice and stick-slip friction proceed in the self-similar mode. Corresponding strain power-law distribution is provided by temperature fluctuations that is much larger than noise intensities of strain and stress. This behavior is fixed by homogenous probability density in which characteristic strain scale is absent. Since the power-type distribution is observed at minor strains it meets self-similar rubbing mode of crystalline ice surface. An analysis of the time dependences of friction force was carried out by using a fast Fourier transform. Fluctuations are detected with the spectral power density of the signal, which is inversely proportional to the frequency and demonstrates the realization of $S_p(\omega) \propto 1/\omega^{\alpha}, 0<\alpha<2$ or "pink" noise. It was found that the behavior of the spectrum is related to the course of the prehistory of nonequilibrium rubbing process. Research of autocorrelation function form of random fluctuations of friction force allowed to reveal the frequency characteristics of rubbing. The presence of weak correlation is shown. The obtained results reflect the real frictional conditions and can be used for predicting the rubbing force value or ice surface states (phases) during a certain correlation time. Thus, it is possible to establish the necessary external conditions to achieve the desired stable ice friction mode.

Liquid layer between tread and ice

T. Kessel, R. Mundl, B. Wies, K. Wiese, Tire Society Meeting, Sept. 2011
C. Klapproth, T. Kessel, K. Wiese, B. Wies, Tribol. Int. **99**, 169, 2016



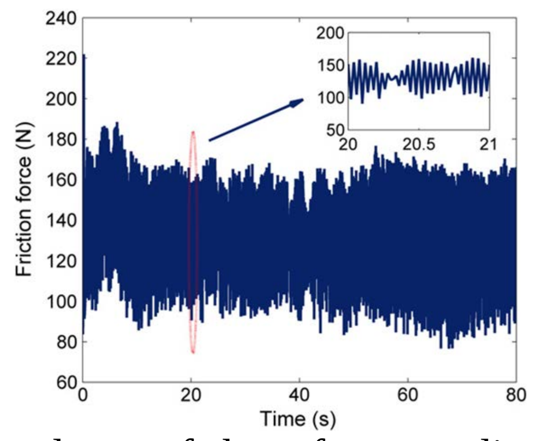
- **Viscous friction in the liquid layer**
- **Ice melting as a result of heating at friction**



Interrupted friction mode - «stick-slip»

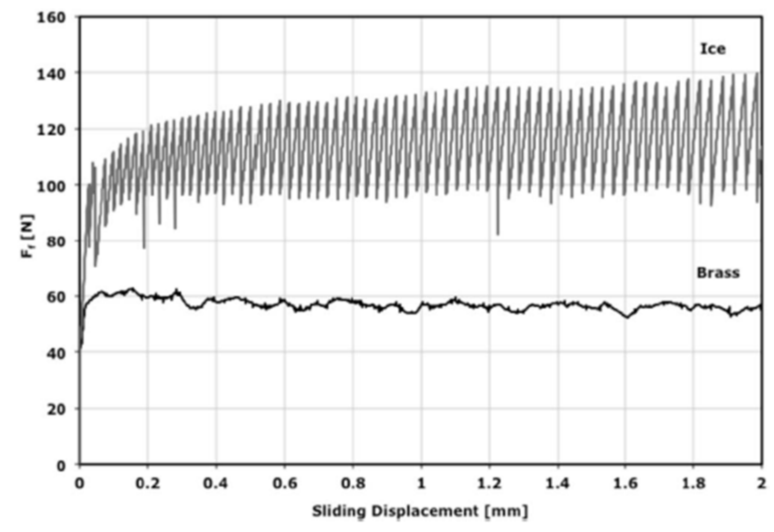
Time dependence of friction force

S. Sukhorukov, S. Løset, Cold Reg. Sci. Technol., 2013, V. 94, 1–12.



Dependence of shear force on displacement

E.M.Schulson, A.L.Fortt, J. Geophys. Res. Solid Earth., 2012, V.117, B12204.



Basic equations

Measure units $\varepsilon_s = \frac{\sigma_s}{G_\varepsilon}$, $\sigma_s = \left(\frac{c_p \eta_\varepsilon T_c}{\tau_T}\right)^{\frac{1}{2}}$, T_c, $(\varepsilon_s/\tau_\varepsilon)^2$, σ_s^2, T_c^2 (1)

for quantities ε, σ, T, I_ε, I_σ and I_T, where $G_\varepsilon \equiv \frac{\eta_\varepsilon}{\tau_\varepsilon} \equiv G(\omega)|_{\omega \to 0}$

Kelvin-Voigt equation for shear strain:

$$\tau_\varepsilon \dot{\varepsilon} = -\varepsilon^a + \sigma + \sqrt{I_\varepsilon}\xi_1(t) \quad (2)$$

Landau-Khalatnikov-type equation for shear stress:

$$\tau_\sigma \dot{\sigma} = -\sigma + g(T-1)\varepsilon^a + \sqrt{I_\sigma}\xi_2(t) \quad (3)$$

Kinetic equation for temperature:

$$\tau_T \dot{T} = (T_e - T) - \sigma\varepsilon^a + \sqrt{I_T}\xi_3(t) \quad (4)$$

$\xi(t)$ is the δ - correlated Gaussian stochastic source (white noise):

$$\langle \xi_i(t) \rangle = 0, \quad \langle \xi_i(t)\xi_j(t') \rangle = 2\delta_{ij}\delta(t-t') \quad (5)$$

A.Khomenko, M. Khomenko, B. Persson, K. Khomenko, Tribology Letters, 2017, V. 65, Iss.2, Art.71.
A. Khomenko, Tribology Letters, 2018, V. 66, Iss.3, Art.82.

Basic equations

Adiabatic approximation without noise $(I_\varepsilon, I_\sigma, I_T = 0, a = 1)$:

$$\tau_\sigma << \tau_\varepsilon, \quad \tau_T << \tau_\varepsilon \quad (6)$$

$\tau_{\varepsilon\,\min} \approx 2\cdot10^{-5}$ s, $\tau_\sigma \approx a/c \sim 10^{-12}$ s, $a \sim 1$ nm, $c \sim 10^3$ m/s

For ice: $\rho \approx 916$ kg/m^3, $\kappa \approx 2.22$ W/m×K, $c_p \approx 2050$ J/kg×K,

$G_\varepsilon \approx 10$ GPa and water at $T = 0^\circ$C: $\eta_\varepsilon \approx 1.8\times10^{-3}$ Pa×s,

the second inequelity (6) $\Rightarrow l << L \Rightarrow$

The maximal distance into which heat penetrates ice:

$$L = \sqrt{\frac{\chi v_\varepsilon}{c_\varepsilon^2}} \approx 10 \text{ nm} \quad (7)$$

Landau-Khalatnikov equation: $\tau_\varepsilon \dot{\varepsilon} = -\partial V / \partial \varepsilon$ (8)

Synergetic potential: $V = \frac{1}{2}\left[\varepsilon^2 + (1-T_e)\ln(1+g\varepsilon^2)\right]$ (9)

Critical temperature: $T_{c0} = 1 + g^{-1}$, $g \equiv G_0/G_\varepsilon < 1$, $G_\varepsilon \equiv \eta_\varepsilon/\tau_\varepsilon$ (10)

Steady-state strain: $\varepsilon_0 = (T_e - (1+g^{-1}))^{1/2}$ (11)

Noise effect

Langevine equation: $\tau_\varepsilon \dot{\varepsilon} = f_a(\varepsilon) + \sqrt{I_a(\varepsilon)}\,\xi(t)$ (12)

Deterministic force: $f_a(\varepsilon) = -\varepsilon^a + g\varepsilon^a (T_e - 1) d_a(\varepsilon)$, (13)
$d_a(\varepsilon) = (1 + g\varepsilon^{2a})^{-1}$

Effective noise intensity:

$$I_a(\varepsilon) \equiv I_\varepsilon + [I_\sigma + I_T(g\varepsilon^a)^2] d_a^2(\varepsilon) \quad (14)$$

Fokker-Planck equation:

$$\frac{\partial P_a(\varepsilon,t)}{\partial t} = -\frac{\partial}{\partial\varepsilon}[f_a(\varepsilon)P_a(\varepsilon,t)] + \frac{\partial}{\partial\varepsilon}\left[\sqrt{I_a(\varepsilon)}\frac{\partial}{\partial\varepsilon}\sqrt{I_a(\varepsilon)}P_a(\varepsilon,t)\right] \quad (15)$$

Stationary distribution:

$$P_a(\varepsilon) = Z^{-1}\exp\{-U_a(\varepsilon)\} \quad (16)$$

Effective potential:

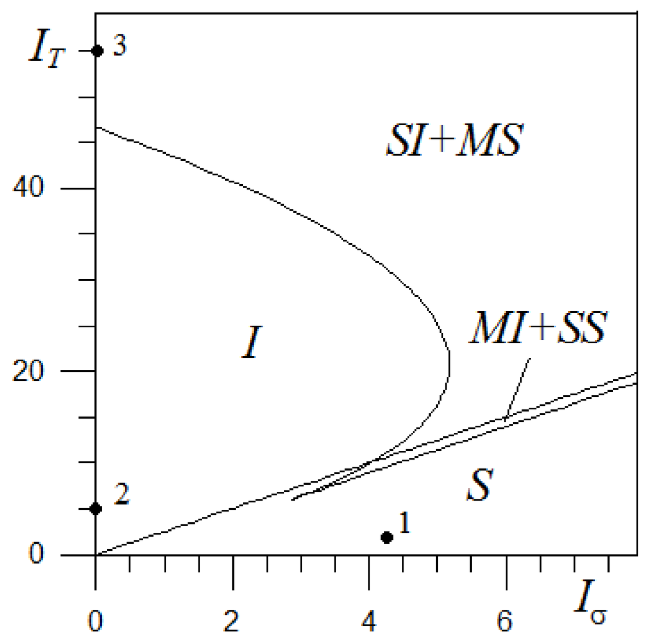
$$U_a(\varepsilon) = \frac{1}{2}\ln I_a(\varepsilon) - \int_0^\varepsilon \frac{fa(\varepsilon')}{I_a(\varepsilon')}d\varepsilon' \quad (17)$$

Extrema of distribution function (16) (potential (17)):

$$d_a^{-3}(\varepsilon) - g(T_e-1)d_a^{-2}(\varepsilon) - ga\varepsilon^{a-1}\{gI_T[d_a^{-1}(\varepsilon) - 2] + 2I_\sigma\} = 0 \quad (18)$$

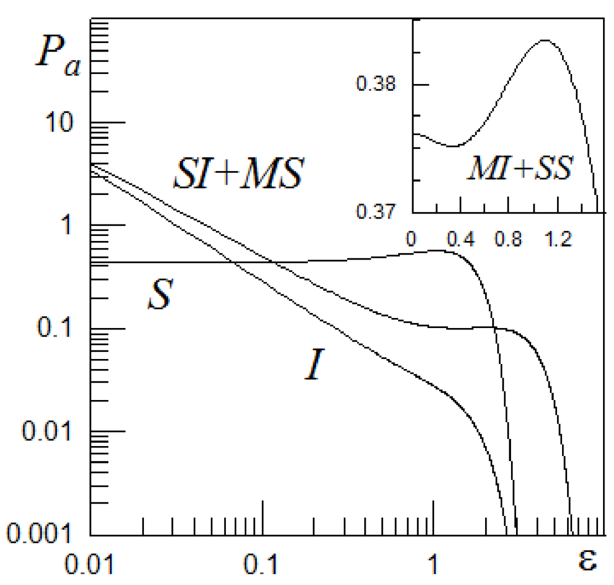
Phase diagram and probability distribution

Phase diagram at $g = 0.8$, $T_e = 1.2$, $a = 0.9$ with domains of softened ice (S), ice (I), stable ice and metastable softened ice ($SI+MS$), metastable ice and stable softened ice ($MI+SS$) (stick-slip)



Existence boundary of ice domain I (zero solution): $I_T = 2I_\sigma/g$ (19)

Probability density at $I_\varepsilon = 0$ and modes shown by points in phase diagram: $1 - I_\sigma = 4.25$, $I_T = 2$ (S); $2 - I_\sigma = 0$, $I_T = 5$ (I); $3 - I_\sigma = 0$, $I_T = 55$ ($SI+MS$); $4 - I_\sigma = 5$, $I_T = 12$ ($MI+SS$)



Time series of friction force

Euler method:

$$\varepsilon_{n+1} = \varepsilon_n + \left(f_a(\varepsilon_n) + \sqrt{I_a(\varepsilon_n)}\frac{\partial}{\partial\varepsilon}\sqrt{I_a(\varepsilon_n)}\right)dt + \sqrt{I_a(\varepsilon_n)}dW_n \quad (20)$$

Box-Muller model: $W_n = \sqrt{\mu^2}\sqrt{-2\ln r_1}\cos(2\pi r_2)$, $r_n \in (0,1]$ (21)

$\langle W_n \rangle = 0$, $\langle W_n W_{n'} \rangle = 0$, $\langle W_n^2 \rangle \to 2$ (22)

For friction of PMMA, rubber, steel and ice on ice:

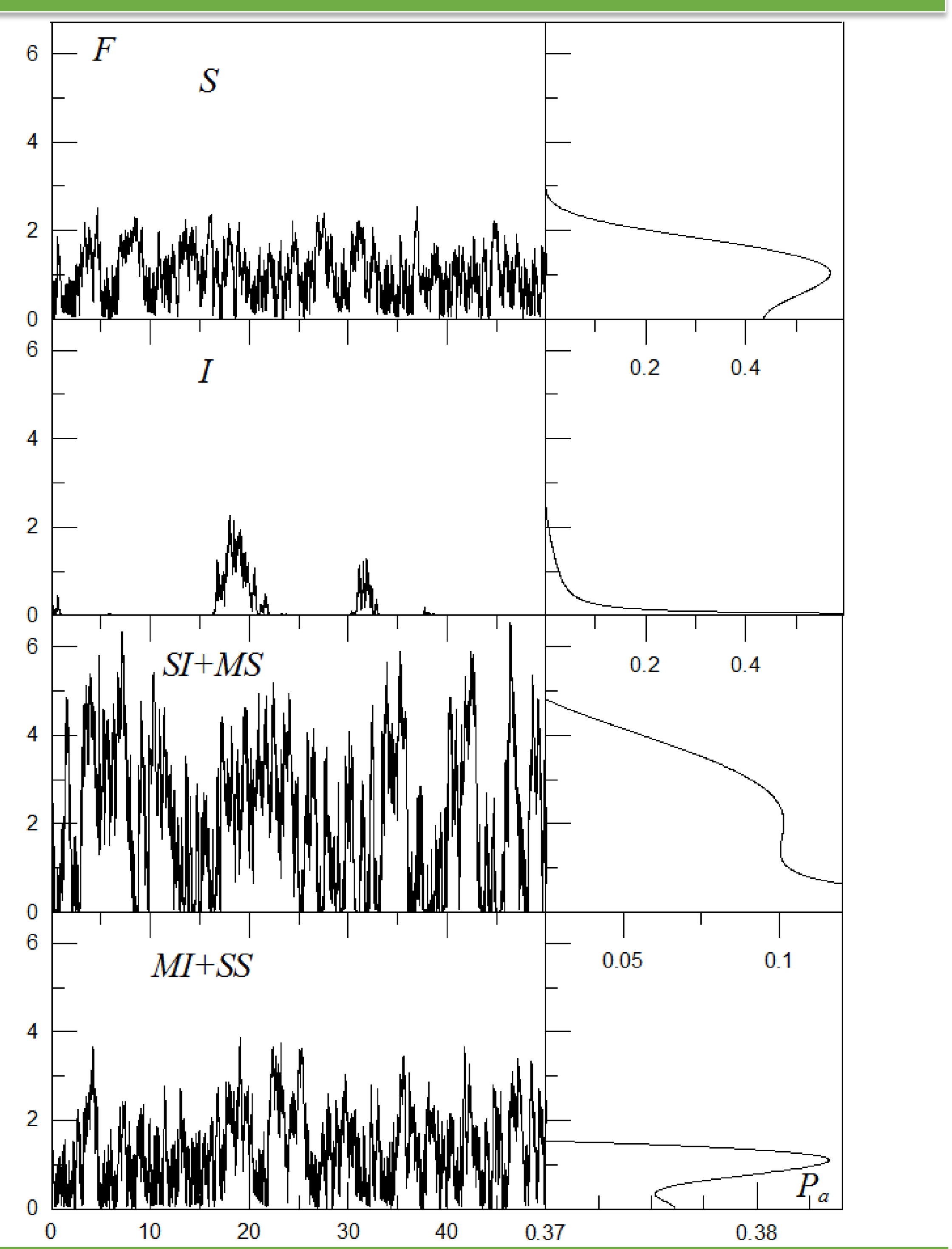
$A \approx 10^{-6} - 10^{-1}$ m^2

Yield strength: $G_\varepsilon \approx 0.1 - 10$ MPa

Relaxation time: $\tau_\varepsilon \approx 0.1 - 5$ s

Friction force: $F(t) = AG_\varepsilon|\varepsilon|(t)$ (23)

Time series of friction force



Conclusions

The generalized pattern is described of the ice surface premelting in terms of concept of self-similar system. Such a position can be implemented in the fractional viscoelastic model, where the strain is the order parameter, the conjugate field is reduced to stress, and the temperature serves as a control parameter. In order to find the self-similarity conditions the nonlinear strain relaxation and feedbacks are introduced in the Lorenz system. Thus, due to correlation analysis the power-law strain distribution inherent in self-similar systems is provoked by increase in temperature fluctuations and fractional exponent. The homogeneous probability density of strain values is observed in its limited interval. In the phase diagram with the *S*, *I*, *SI+MS* and *MI+SS* regions the domain of crystalline ice friction is limited by comparatively small thermostat temperatures and noises intensities of the stress and temperature. The new region *MI+SS* of metastable ice and stable softening (*stick-slip* rubbing) appears. Friction force time series have been calculated for each mode numerically using the Langevin equation, and it has been concluded that at determined parameters time series are self-similar. In agreement with above examination self-similar rubbing force time dependencies are realized only for the crystalline ice friction (*I*) and *SI+MI stick-slip* domains, since only for these modes power-law probability density is revealed. Fourier and autocorrelation function analysis show the colored nature of noise and presence of time correlation that predicts the possibility to govern ice friction.