The stability analysis of stationary modes Of the ice surface softening during the friction process

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Introduction

In this paper, a further study of synergetic model of ice softening during friction is carried out. The origin model describes softening of the ice surface in consequence of spontaneous appearance of shear strain due to external supercritical heating. The result is a self-organization process of the strain (order parameter), stress shear components (conjugate field) and layer temperature (control parameter). In this case, the Kelvin-Voigt equation for viscoelastic medium, equation for heat conductivity and relaxation equation of Landau-Khalatnikov-type were taken as a basis of consideration [1].

Atomic models of ice friction have recently emerged. If we use the Ginzburg-Landau free energy in the first-order phase transition and the molecular dynamics method, we see that the film on the melted ice surface will previously consist of some molecular layers, and its thickness will increase with stress and temperature. This leads to a decrease in friction due to an increase in the lubricant layer due to the weakening of the hydrogen bonds between the ice molecules. The dependence of the friction force on the speed increases linearly due to the viscous component of the stress that occurs in the liquid film on the ice surface during shear. Over time, there is a relaxation of the shear stress component/

The basis of our study is the softening of ice during friction, which is provided on the one hand by self-organization of stress components σ and deformation ε, and on the other by temperature T. We know [1] the relationship between stress components σ and deformation ε, the Kelvin - Voigt model describes the simplest case, and the temperature effect is caused by critical increase of the shear modulus G (T) at the reduced temperature: G = 0 for water and G ≠ 0 for ice.

Methods

The model we take as a basis [1] consists of two planes: it is the friction of ice with some material, or the friction of ice against the surface of ice, which are separated by a lubricating softened layer of ice.

The kinetic equation for temperature T is obtained using the relation of the theory of elasticity. As a result, the mathematical model of the melting process is given by a system of differential equations

$$\begin{cases} \dot{\varepsilon} = -\dfrac{\varepsilon}{\tau_\varepsilon} + \dfrac{\sigma}{\eta_\varepsilon}, & (1) \\ \tau_\sigma \dot{\sigma} = -\sigma + G(T)\varepsilon, & (2) \\ C_p \dot{T} = k\nabla^2 T - \sigma\dfrac{\varepsilon}{\tau_\varepsilon} + \dfrac{\sigma^2}{\eta_\varepsilon}. & (3) \end{cases}$$

In equation (1) τ_ε is the Debye relaxation time, η_ε is the effective shear viscosity coefficient. The second term of the right part describes the flow of viscous fluid due to the shear component of the stress σ.

Then the basic equations (1), (2) and (3) to describe the behavior of a viscoelastic medium in dimensionless form will be

$$\begin{cases} \dfrac{d\varepsilon}{dt} = -\varepsilon + \sigma, & (4) \\ \alpha\dfrac{d\sigma}{dt} = -\sigma + g(T-1)\varepsilon, & (5) \\ \beta\dfrac{dT}{dt} = (T_e - T) - \sigma\varepsilon. & (6) \end{cases}$$

It is constant here

$g = G_0 / G_\varepsilon$.

The system (4) - (6) is qualitative and can describe various features of the boundary mode.

Equations (4), (5), (6) have a form similar to the Lorentz scheme, which allows us to denote the thermodynamic phase and kinetic transitions.

Phase portraits

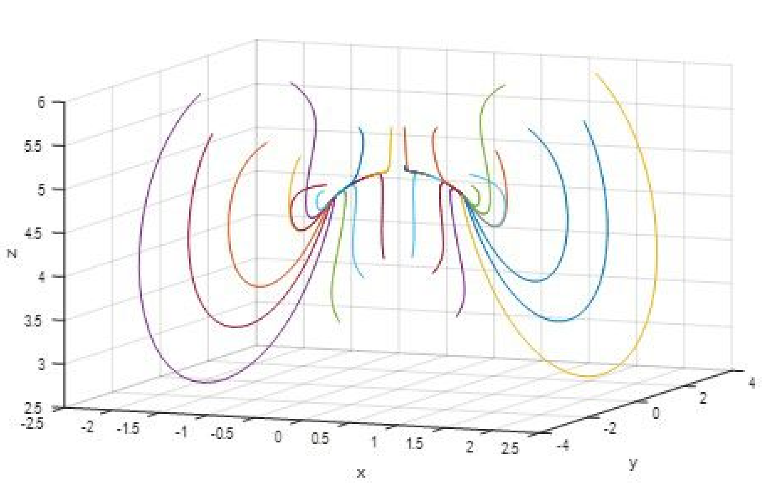

Figure 1. Phase portraits for the 1st region for α = 4, β = 5, g = 0.27, T_e = 5 three-dimensional portrait with coordinates x, y, z

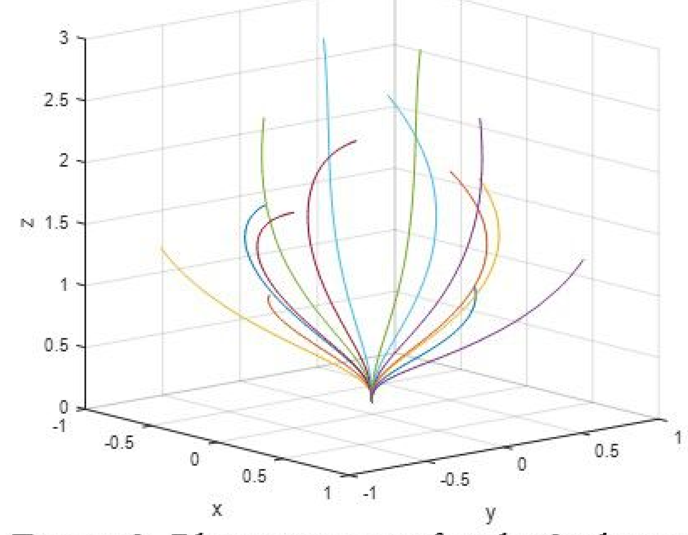

Figure 2. Phase portraits for the 2nd region for α = 4, β = 5, g = 0.8, T_e = 0.1 three-dimensional portrait with coordinates x, y, z

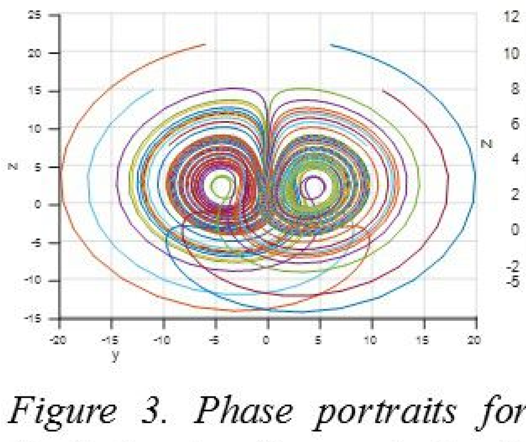

Figure 3. Phase portraits for the 3rd region for α = 4, β = 5, g = 0.88, T_e = 20 three-dimensional portrait with coordinates x, y, z

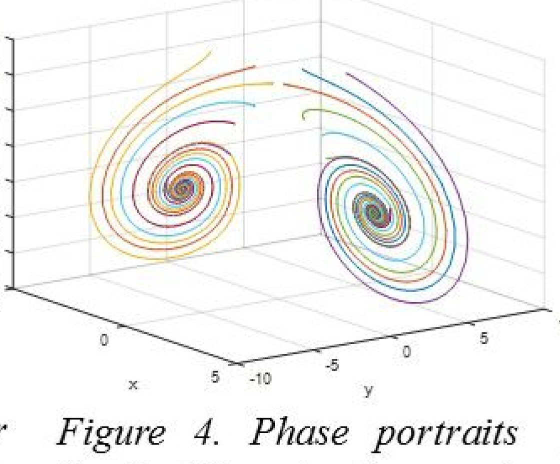

Figure 4. Phase portraits for the 4th region for α = 4, β = 5, g = 0.4, T_e = 10 three-dimensional portrait with coordinates x, y, z

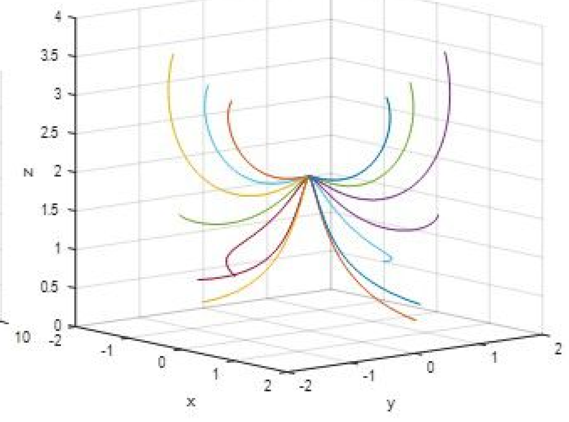

Figure 5. Phase portraits for the 5th region for α = 4, β = 5, g = 0.8, T_e = 0.1 three-dimensional portrait with x, y, z

Steady states of the system

To determine the stable states of the system in terms of the phase plane method, it is necessary to find the coordinates of the singular points.

$$\varepsilon = \pm\sqrt{T_e - 1 - \frac{1}{g}},$$
$$\sigma = \pm\sqrt{T_e - 1 - \frac{1}{g}},$$
$$T = 1 + \frac{1}{g}.$$

"+" - corresponds to point A, "-" – to the point B, T - the same for all.

For point C we have the following coordinates $\sigma_0 = 0, \varepsilon_0 = 0, T_0 = T_e$.

Table 1 - Values of coordinates and indicators of Lyapunov for point C depending on the parameters α, β, g, T_e.

Set of parameters		1	2	3	4	5
Parameters	a	4	4	4	4	4
	b	5	5	5	5	5
	g	0,27	0,8	0,88	0,4	0,4
	T_e	5	0,1	20	10	2
Coordinates	s_1	0	0	0	0	0
	ε_1	0	0	0	0	0
	T_1	5	0,1	20	10	2
Lyapunov indicators	λ_{11}	-0,2	-0,2	-0,2	-0,2	-0,2
	λ_{12}	0,015800281	-0,625+0,19i	1,453611315	0,395110288	-0,134464578
	λ_{13}	-1,265800281	-0,625-0,19i	-2,703611315	-1,645110288	-1,115535422
Types of stability of singular points		Saddle-stable knot	Stable knot-focus	Saddle-stable knot	Saddle-stable knot	Stable knot

Table 2 - Values of coordinates and indicators of Lyapunov for point A depending on the parameters α, β, g, T_e.

Set of parameters		1	2	3	4	5
Parameters	a	4	4	4	4	4
	b	5	5	5	5	5
	g	0,27	0,8	0,88	0,4	0,4
	T_e	5	0,1	20	10	2
Coordinates	s_2	0,544331054	–	4,226539526	2,549509757	-
	ε_2	0,544331054	–	4,226539526	2,549509757	-
	T_2	4,703703704	2,25	2,136363636	3,5	3,5
Lyapunov indicators	λ_{21}	-1,2523	-1,1911	-1,4718	-1,3114	-1,2319
	λ_{22}	-0,1571	-0,5309	0,0109 + 1,0334i	-0,0693 + 0,4398i	-0,3552
	λ_{23}	-0,0407	0,2720	0,0109 – 1,0334i	-0,0693 – 0,4398i	0,1371
Types of stability of singular points		Stable knot	–	Focus of complex architecture	Stable knot-focus	–

Table 3 - Values of coordinates and indicators of Lyapunov for point B depending on the parameters α, β, g, T_e.

Set of parameters		1	2	3	4	5
Parameters	a	4	4	4	4	4
	b	5	5	5	5	5
	g	0,27	0,8	0,88	0,4	0,4
	T_e	5	0,1	20	10	2
Coordinates	s_2	-0,544331054	–	-4,226539526	-2,549509757	-
	ε_2	-0,544331054	–	-4,226539526	-2,549509757	-
	T_2	4,703703704	2,25	2,136363636	3,5	3,5
Lyapunov indicators	λ_{21}	-1,2523	-1,1911	-1,4718	-1,3114	-1,2319
	λ_{22}	-0,1571	-0,5309	0,0109 + 1,0334i	-0,0693 + 0,4398i	-0,3552
	λ_{23}	-0,0407	0,2720	0,0109 – 1,0334i	-0,0693 – 0,4398i	0,1371
Types of stability of singular points		Stable knot	–	Focus of complex architecture	Stable knot-focus	–

Conclusions

The paper considers a mathematical model of the process of melting an ultrathin layer of ice between two solid smooth surfaces. It was believed that the melting of this layer is possible because of heating and the action of shear deformation. The latter was described by the Voigt-Kelvin equation for a viscoelastic medium. For the control parameter - temperature - the relaxation equation of thermal conductivity was used, for the conjugate field - stress - the equation of Landau-Khalatnikov type.

The result is a system of three differential equations, the analysis of which was performed by the phase plane method. The coordinates of the steady states of the system and the corresponding Lyapunov indicators for all possible modes of realization of the solid and softened state of a thin layer of ice were determined. For each of these modes, based on phase portraits, the kinetics of the ice melting process was analyzed depending on external parameters

References

1. *Khomenko A.V., Khomenko K.P., FalkoV.V.* Nonlinear model of ice surface softening during friction // Condens. Matter Phys.-2016.-**19.**-P. 33002: 1–10 .