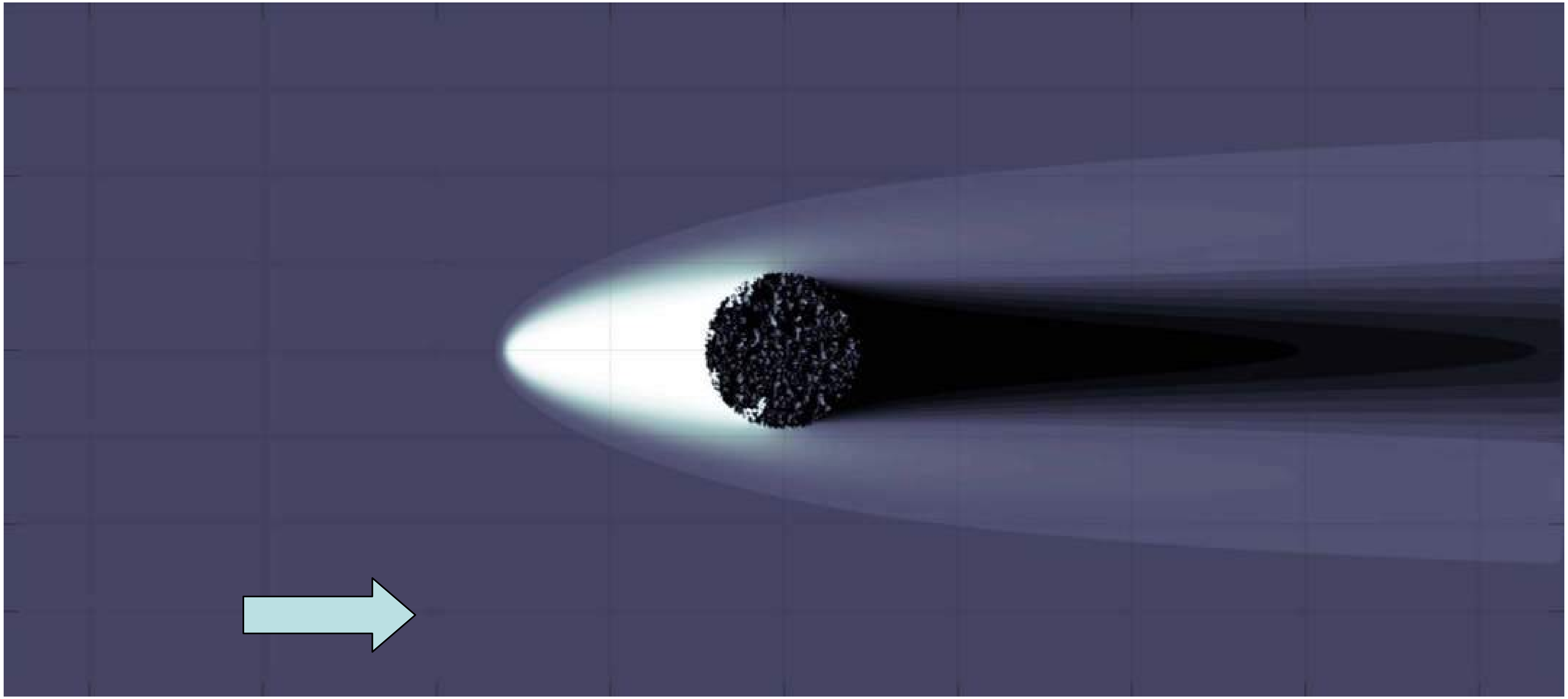
Effects of dissipative pairing, wake inversion and sign change of effective dissipative forces between impurities in gas flow

Role of short-range interaction and non-linear blockade effect

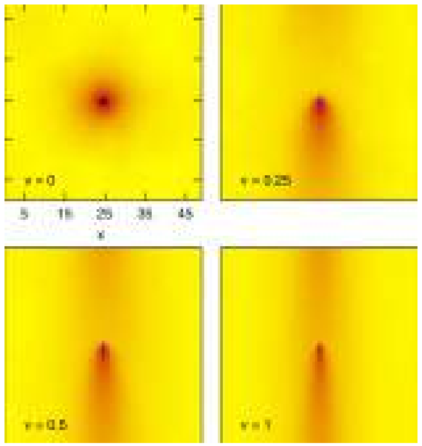
O.V. Kliushnychenko · S.P. Lukyanets

Department of Theoretical Physics

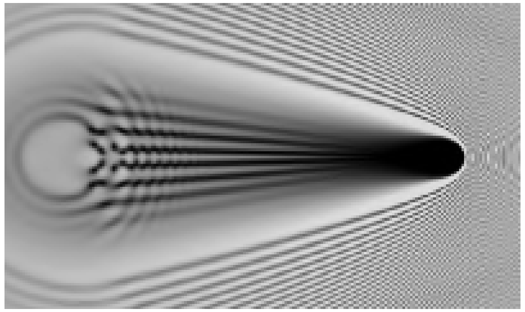


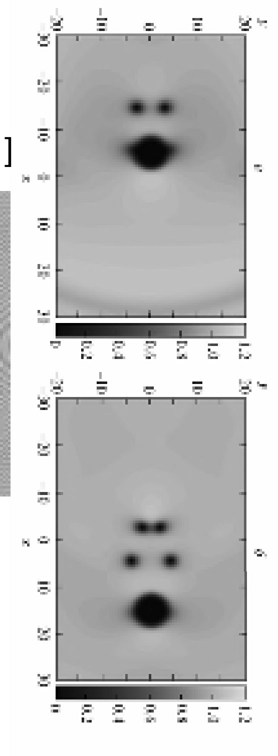
Wakes and wake-mediated interaction of various physical nature

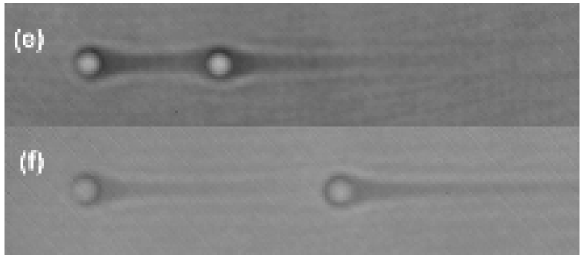
Wake of magnetic impurity moving through **ferromagnet** [PRL **104**, 080601 (2010)]



Cherenkov radiation and vortices [JETP **110**, 877 (2010)]

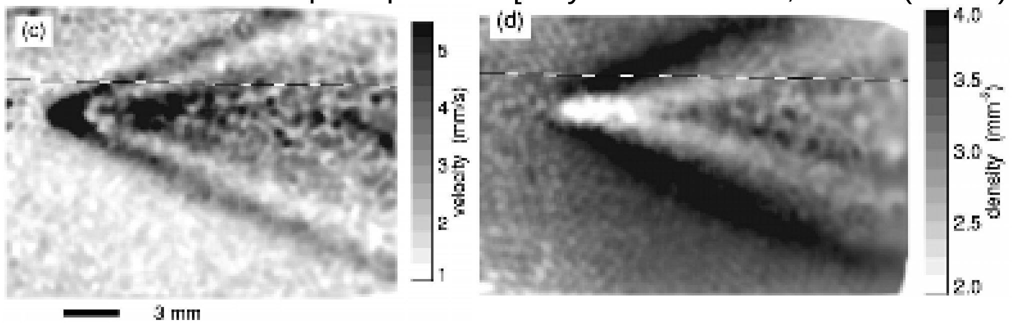




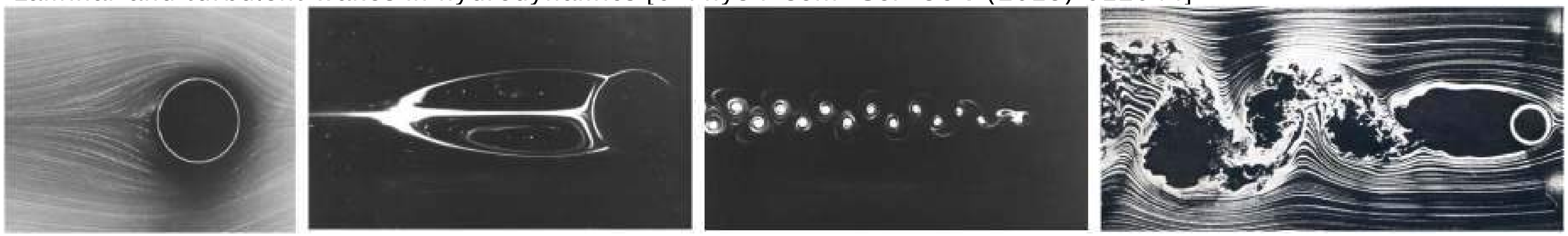


Non-equilibrium depletion forces **in colloidal suspension** [PRE **91**, 042303 (2015)]

Mach cones in complex plasma [Phys. Rev. E **61**, 5557 (2000)]



Laminar and turbulent wakes in hydrodynamics [J. Phys.: Conf. Ser. **594** (2015) 012044]



Properties of diffusive wakes and wake-mediated interaction

1) large spatial extension $R^{\text{int}}_{eq(elastic)} << R^{\text{int}}_{noneq}$

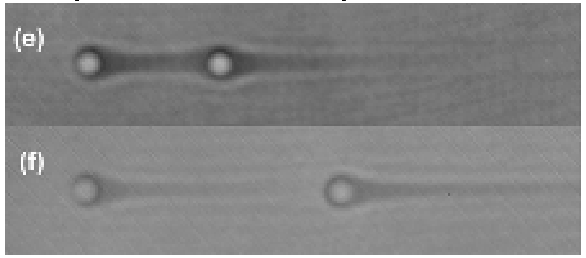
   $\rightarrow$ memory (long-time correlations)

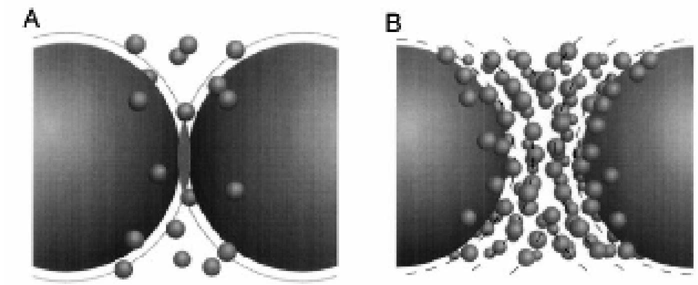
2) long-range interaction

3) anisotropy $F_{12} \neq - F_{21}$

   (non-Newtonian interaction)

non-equilibrium depletion force





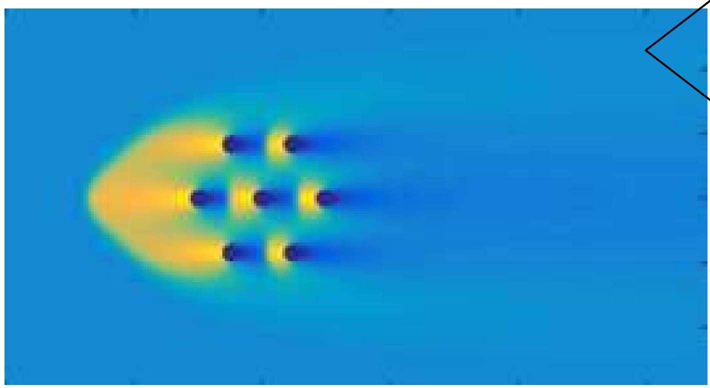
equilibrium depletion force

short-range inter-particle interaction in Brownian gas (lattice gas model)

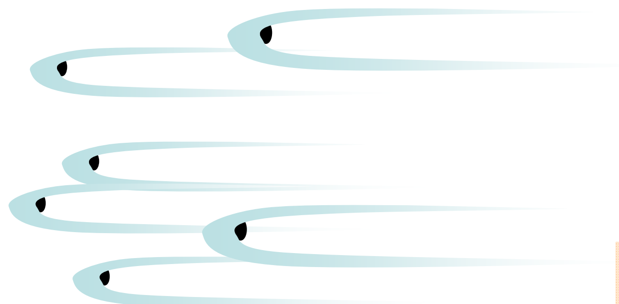
changes in structure of wakes *(blockade effects)*

character of dissipative interaction

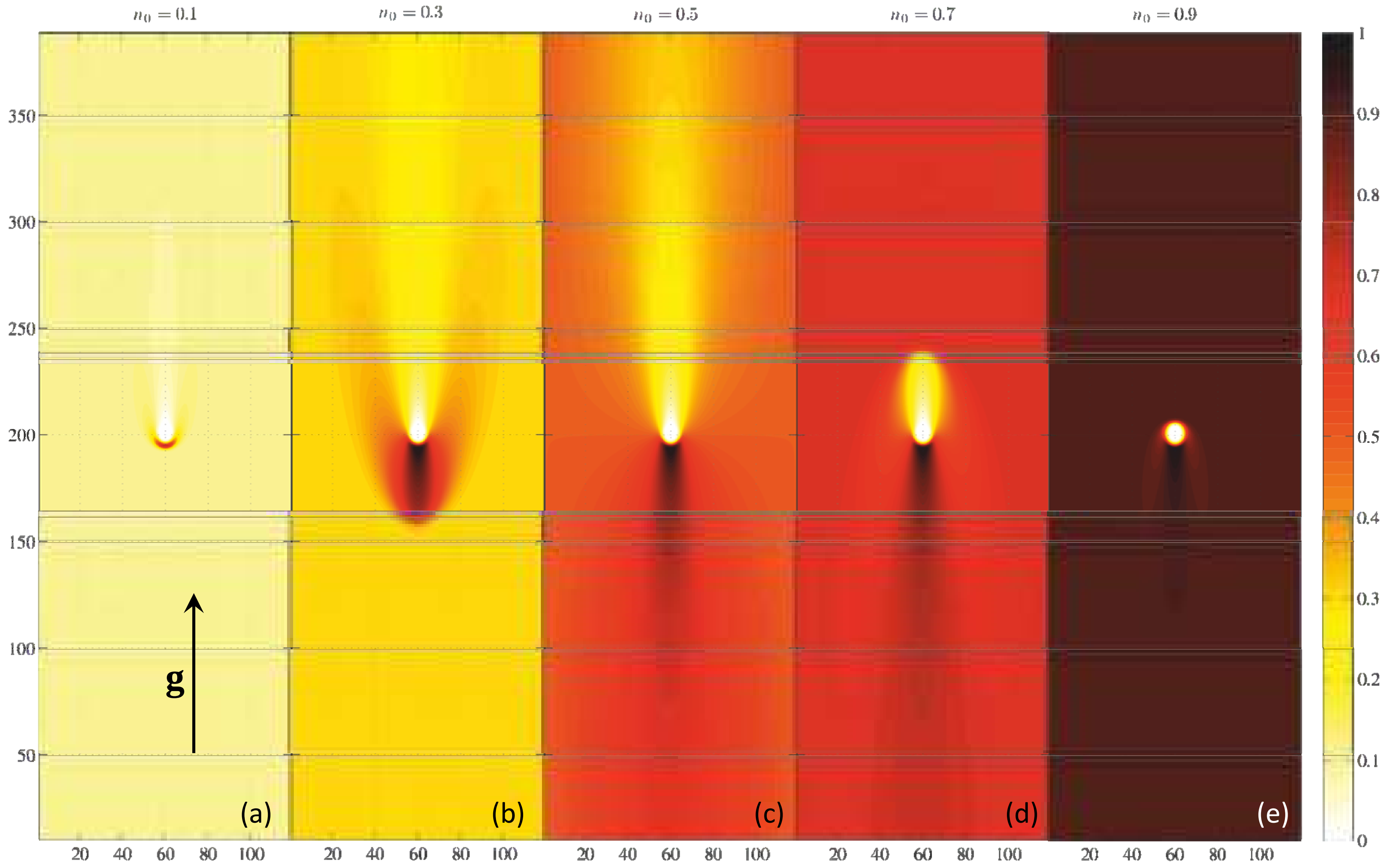
non-linear blockade effects



blockade effects can be neglected



Effect of concentration-dependent wake inversion



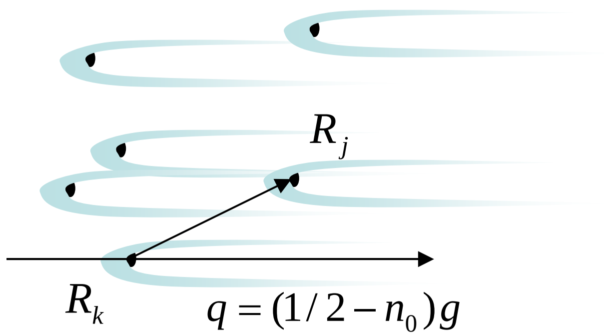
Concentration distribution of the lattice gas particles n(x,y) around obstacle embedded into flow (2D)
n_0 is bath fraction

The type of dissipative interaction:

Induced dipole-dipole (multipole) interaction, associated with anisotropic (asymmetrical) Yukawa potential, total "induced charge" is not conserved

---

$$\nabla^2 n - U\nabla^2 n + n\nabla^2 U - (\mathbf{g}\cdot\nabla)n(1-n-U) = 0,$$



$$\boxed{\begin{aligned} &\delta n(\mathbf{r}) \sim (\overline{\mathbf{\Omega}}\cdot\nabla_{\mathbf{r}})G(\mathbf{r}), \\ &f_{ki} \sim -\nabla_{\mathbf{R}_k}\left(\overline{\mathbf{\Omega}}_i\cdot\nabla_{\mathbf{R}_k}\right)G(\mathbf{R}_k - \mathbf{R}_i) \end{aligned}}$$

$$G(\mathbf{r}) = \frac{1}{2\pi}e^{\mathbf{q}\cdot\mathbf{r}}K_0(q|\mathbf{r}|)$$

2D

$$\boxed{G(\mathbf{r}) = \frac{1}{4\pi}\,\frac{e^{-q|\mathbf{r}|+\mathbf{q}\cdot\mathbf{r}}}{|\mathbf{r}|}}$$

3D

$$\underline{\mathbf{q} = (1/2 - n_0)\,\mathbf{g}}$$

Allows to describe:

- Wake asymmetry
- Wake inversion and switching of dissipative interaction
- Non-Newtonian character of dissipative forces $f_{ki} \neq -f_{ik}$

A more rigorous analytical approach: single-layer potential method

$$\nabla \cdot \{\varepsilon[\nabla\psi - \psi(1-\psi)\mathbf{g}]\} = 0, \qquad \nabla \cdot [\varepsilon(\nabla\delta\psi - 2\mathbf{q}\delta\psi - \mathbf{Q})] = 0,$$

$$\varepsilon = \varepsilon(\mathbf{r}) = [1-u(\mathbf{r})]^2$$

$$\psi(\mathbf{r}) = n(\mathbf{r})/[1-u(\mathbf{r})]$$

$$\delta\psi(\mathbf{r}) = \sum_i \int_{S_i} G(\mathbf{r}-\mathbf{r}')\mu_i(\mathbf{r}')\,d\mathbf{r}',$$

$$2\lambda_i[\nabla_{\mathbf{n}}^{+} - 2q_n(\mathbf{r}_i)]\sum_j \int_{S_j} G(\mathbf{r}_i-\mathbf{r}_j)\mu_j(\mathbf{r}_j)\,d\mathbf{r}_j$$

$$+(\lambda_i - 1)\mu_i(\mathbf{r}_i) = 2\lambda_i Q_n(\mathbf{r}_i),$$

nonlinear collective effects

Agreement with numerical calculations within LG model in the mean-field approx.

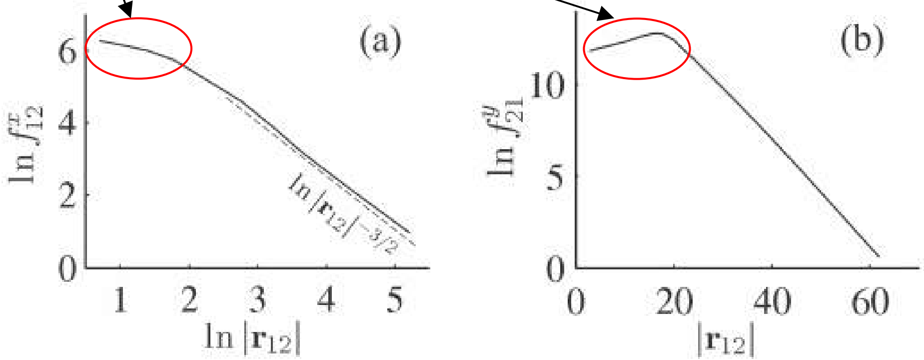


FIG. 8. Asymptotic behavior of dissipative forces (a) f_{12}^x (longitudinal alignment) and (b) f_{12}^y (transverse alignment) at large interobstacle separation r_{12}. The slope on (a) corresponds to the asymptotics $f_{12}^x \sim r_{12}^{-3/2}$. Equilibrium concentration $n_0 = 0.8$, external field $\mathbf{g}$ ($|\mathbf{g}| = 0.5$) is directed along the x axis, the impermeable circular obstacles are of radius $a = 7$ (in units of ℓ), forces are in units of kT/ℓ.

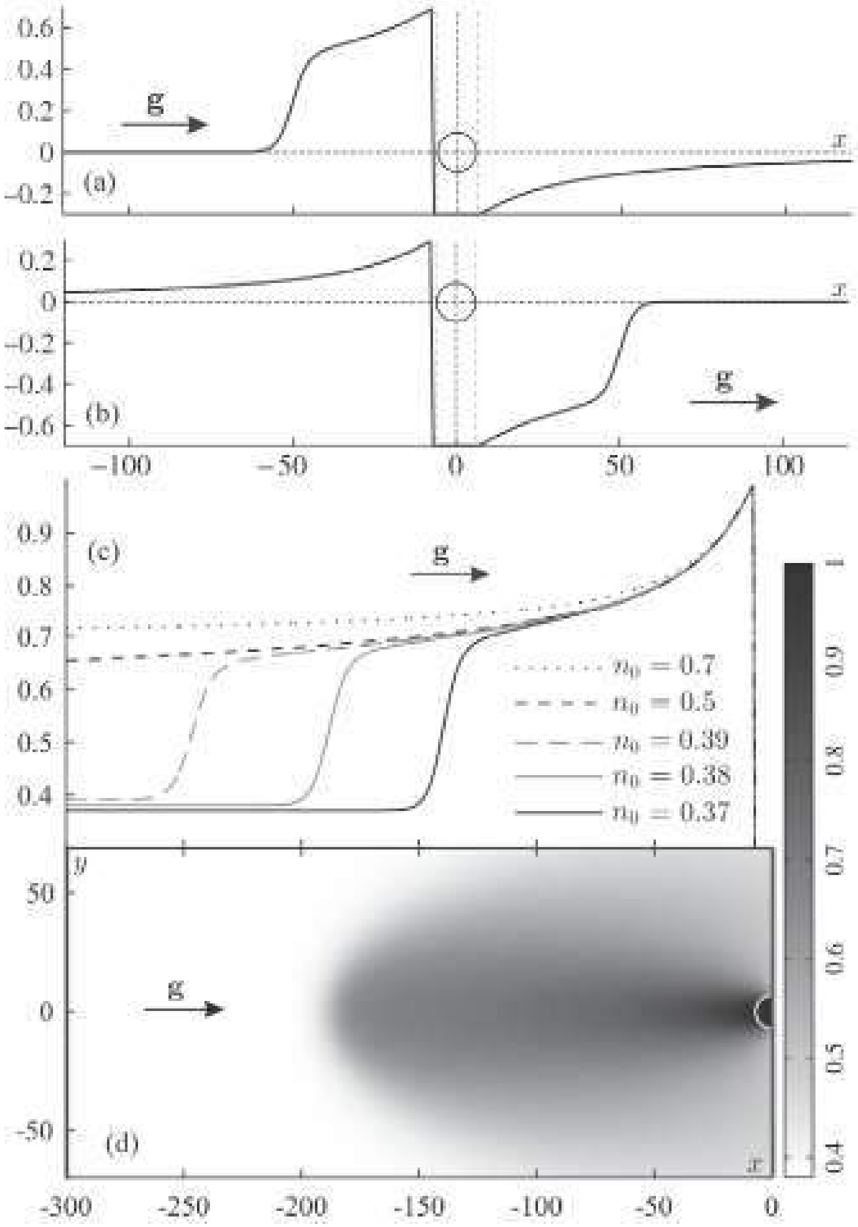
$$\delta n(\mathbf{r}) = \sum_j \int_{S_j} G(\mathbf{r}-\mathbf{r}_j)\mu_j(\mathbf{r}_j)\,d\mathbf{r}_j,$$

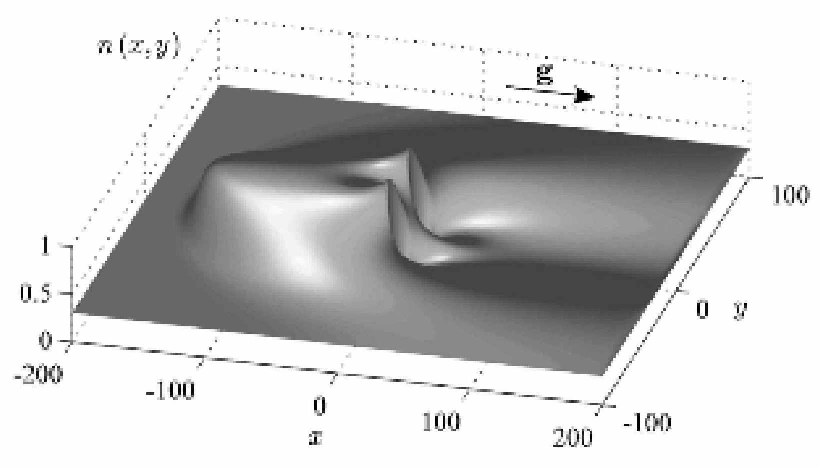
$$\boldsymbol{f}_k = -\sum_j \int_{S_k}\int_{S_j} \mathbf{n}(\mathbf{r}_k)G(\mathbf{r}_k-\mathbf{r}_j)\mu_j(\mathbf{r}_j)\,d\mathbf{r}_k d\mathbf{r}_j.$$

$$\delta n(\mathbf{r}) \approx \frac{e^{\mathbf{q}\cdot\mathbf{r}-q|\mathbf{r}|}}{|\mathbf{r}|}\tilde{I}(\mathbf{r},\mathbf{q})$$

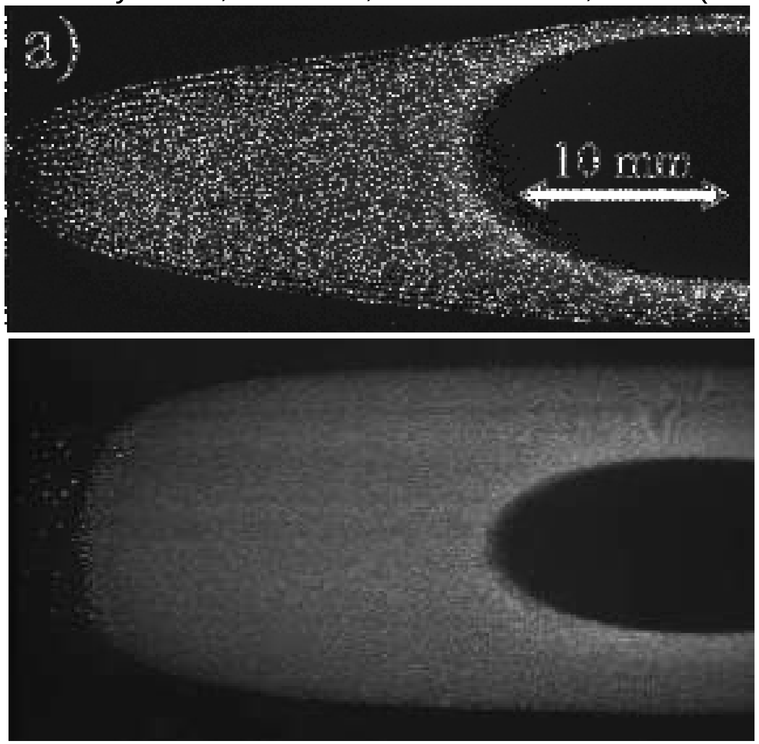
$$\boldsymbol{f}_{ki} \propto -\frac{e^{-q|\mathbf{r}_{ki}|+\mathbf{q}\cdot\mathbf{r}}}{|\mathbf{r}|^{1/2}}\int_{S_k} \mathbf{n}(\mathbf{x}_k)I(\mathbf{r}_{ki},\mathbf{q},\mathbf{x}_k)\,d\mathbf{x}_k.$$

Non-linear blockade effect and shock-wave formation of kink-like distribution profile near big and closely located obstacles

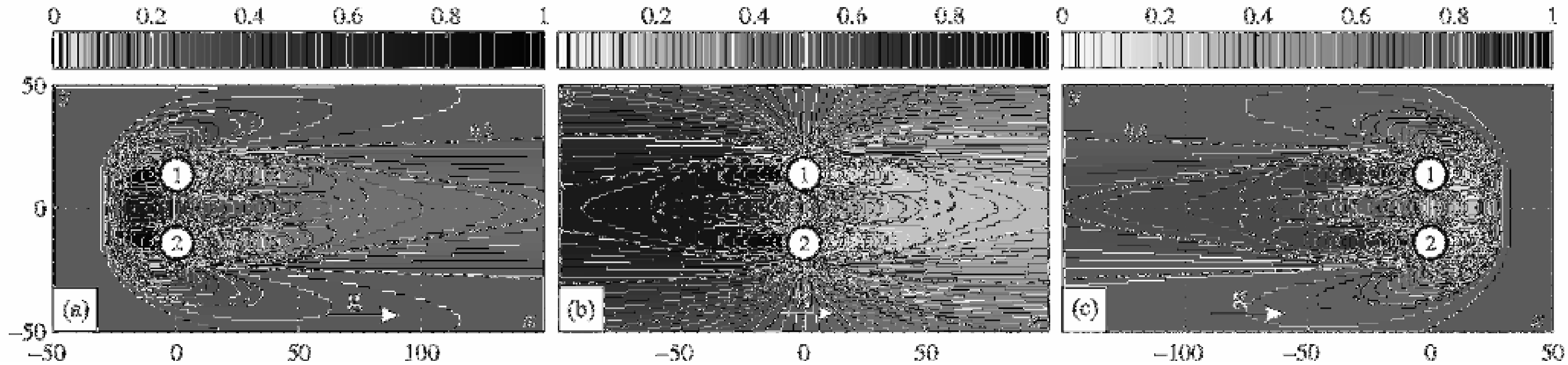




Formation of compact structures in dusty/complex plasma
[New J. Phys. **10**, 033036; M. Schwabe, Diss. (2009)]

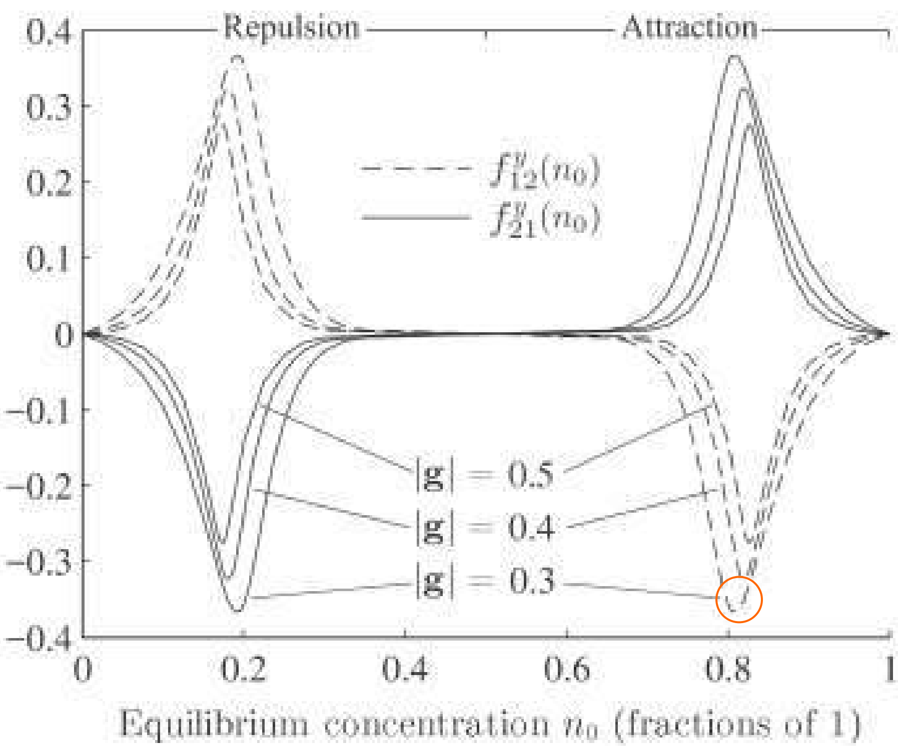


Effect of switching of dissipative interaction (numerical results)



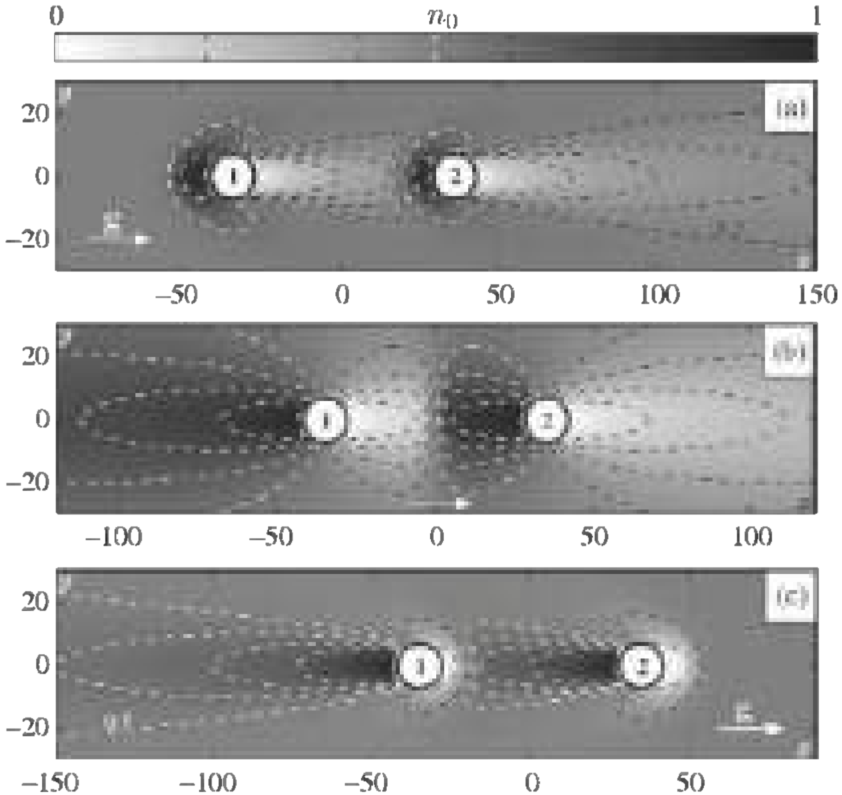
$n_0 = 0.2$ $n_0 = 0.5$ $n_0 = 0.8$

Steady-state concentration distributions (average occupation numbers) of the gas particles n(x,y) near the obstacles, evaluated numerically within the mean-field approx.

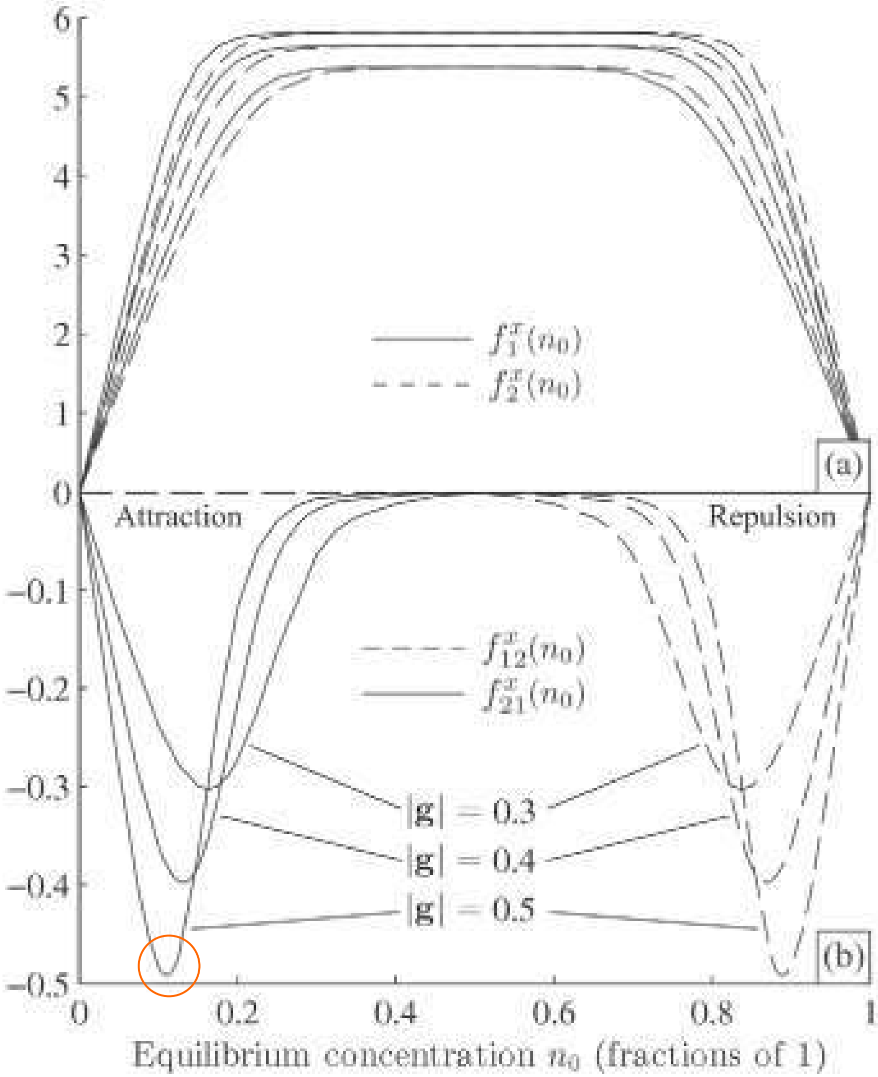


Dependencies of y-components of dissipative forces against gas concentration.

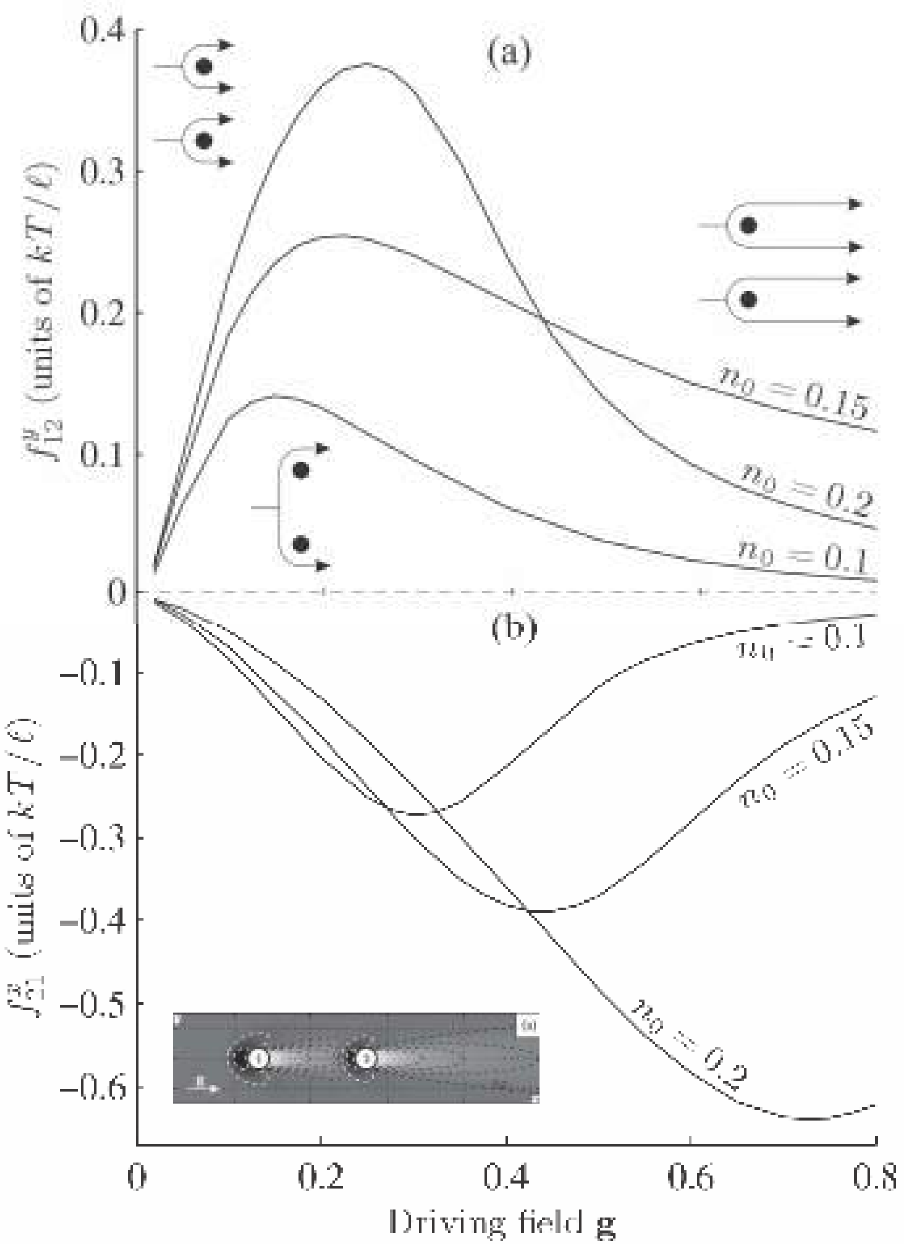
Result: We show the possibility of concentration-dependent switching of dissipative interaction between inclusions (from attraction to repulsion or vice-versa) due to wake inversion effect, entailed by the nonlinear blockade effect for the gas flow of Brownian particles with short-range interaction.



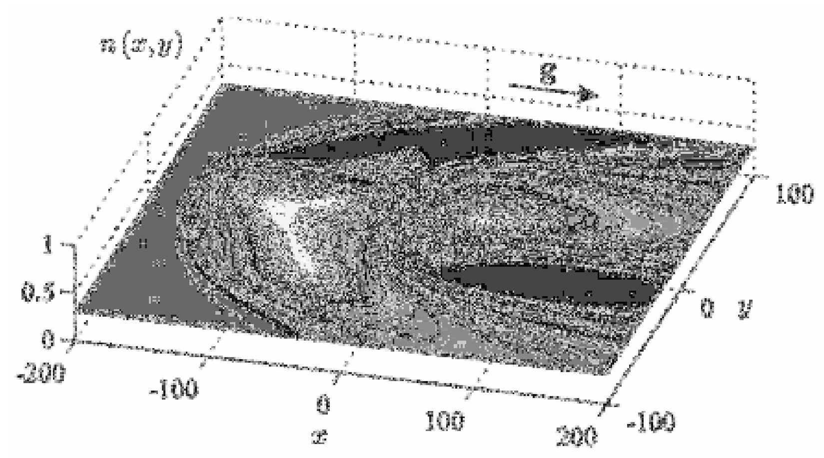
Steady-state concentration distributions (average occupation numbers) of the gas particles n(x,y) near the obstacles, evaluated numerically within the mean-field approx.



Dependencies of dissipative forces against gas concentration.



Result: The formation of common perturbation “coat” (or common wake) around obstacles and non-linear blockade effect can lead to significant enhancement of the effective dissipative interaction for closely located inclusions and, as a result, to the effect of dissipative pairing, i.e., the formation of quasi-inclusion.



Dependencies of dissipative forces
f^y_{12} (a) for orthogonal spatial alignment and
f^x_{21} (b) longitudinal one
against magnitude of external driving field
at bath fractions 0.1, 0.15, 0.2.

Main results

- The effect of concentration-dependent switching (attraction-repulsion) of the effective dissipative interaction between obstacles due to wake inversion entailed by the nonlinear blockade effect for a gas flow of Brownian particles with short-range interaction is shown.

- It is established, that for small and/or distant inclusions the dissipative (wake-mediated) interaction is a kind of dipole-dipole (generally, multipole) interaction associated with anisotropic Yukawa potential and, formally, belongs to the induced dipole-dipole interaction with nonconservation of total induced charge. The latter entails known non-Newtonian behavior of dissipative forces. To this aim, the single-layer potential approach was generalized to the non-equilibrium case.

- It is shown, for the case of closely located inclusions, that dissipative interaction is determined by the non-linear blockade effect which leads to additional enhancement of the gas flow screening. This results in essential enhancement of dissipative interaction and can lead to the dissipative paring effect between inclusions. The latter is manifested by generation of pronounced step-like density profile due to formation of common non-linear perturbation “coat” around inclusions.

- All the results are obtained within classical lattice gas model in the mean-field approximation and are in qualitative agreement with experiments on structure formation in dusty plasmas and colloidal suspensions.

[1] O.V. Kliushnychenko, S.P. Lukyanets, Phys. Rev. E **95**, 012150 (2017)
[2] S.P. Lukyanets, O.V. Kliushnychenko, *Blockade effect and switching of non-equilibrium depletion forces …*, Bogolyubov Conference Problems of Theoretical Physics. — Kyiv, 2016.
[3] O.V. Kliushnychenko, S.P. Lukyanets, Preprint arXiv: 1507.06914v3, (2016)

Applications

Developed approaches and obtained results may be of interest when considering the dissipative structure formation, collective friction force or collective energy losses in an ensemble of inclusions.

In particular, the results can find applications

- in systems with driven hopping transport:
    - surface kinetics of adsorbed atoms
    - superionic conductors

- can serve as a rough model of structure formation in
    - dusty/complex plasmas
    - granular gases
    - colloidal dispersions
    - under motion of magnetic impurities
    - two-dimensional electron plasma in the presence of defects.